Superconductivity and Mottness: Exact Results

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with N&V by J. Zaanen

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Luke Yeo | Edwin Huang | G. La Nave

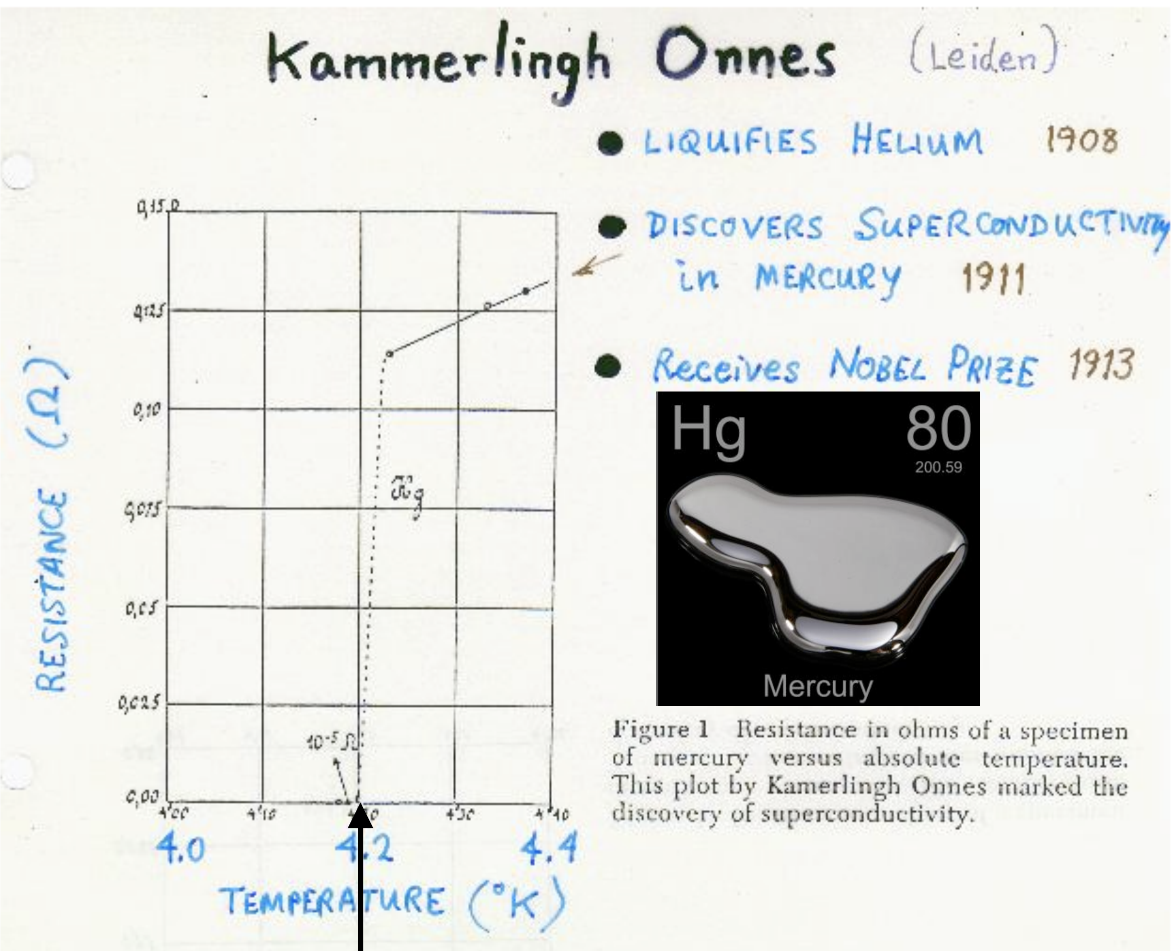
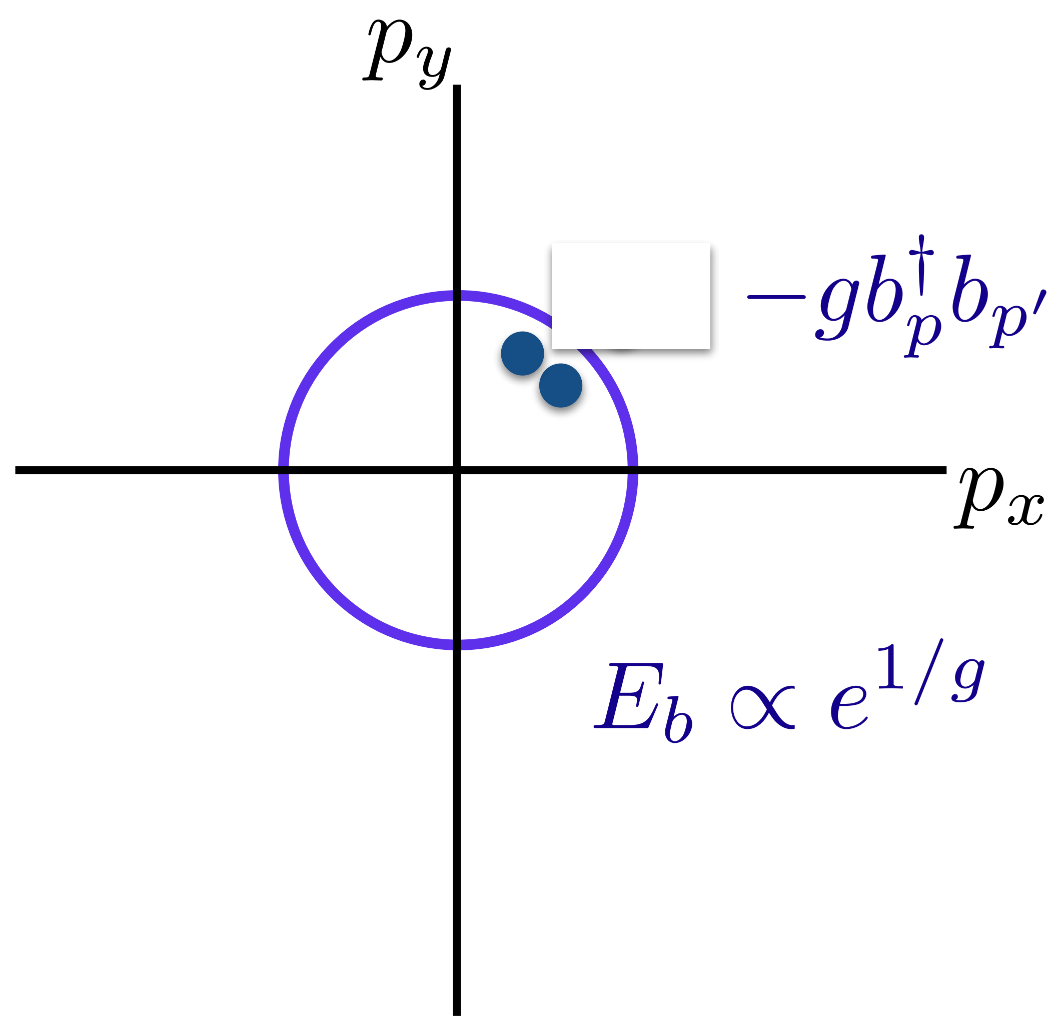


Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.

T_c

Cooper instability

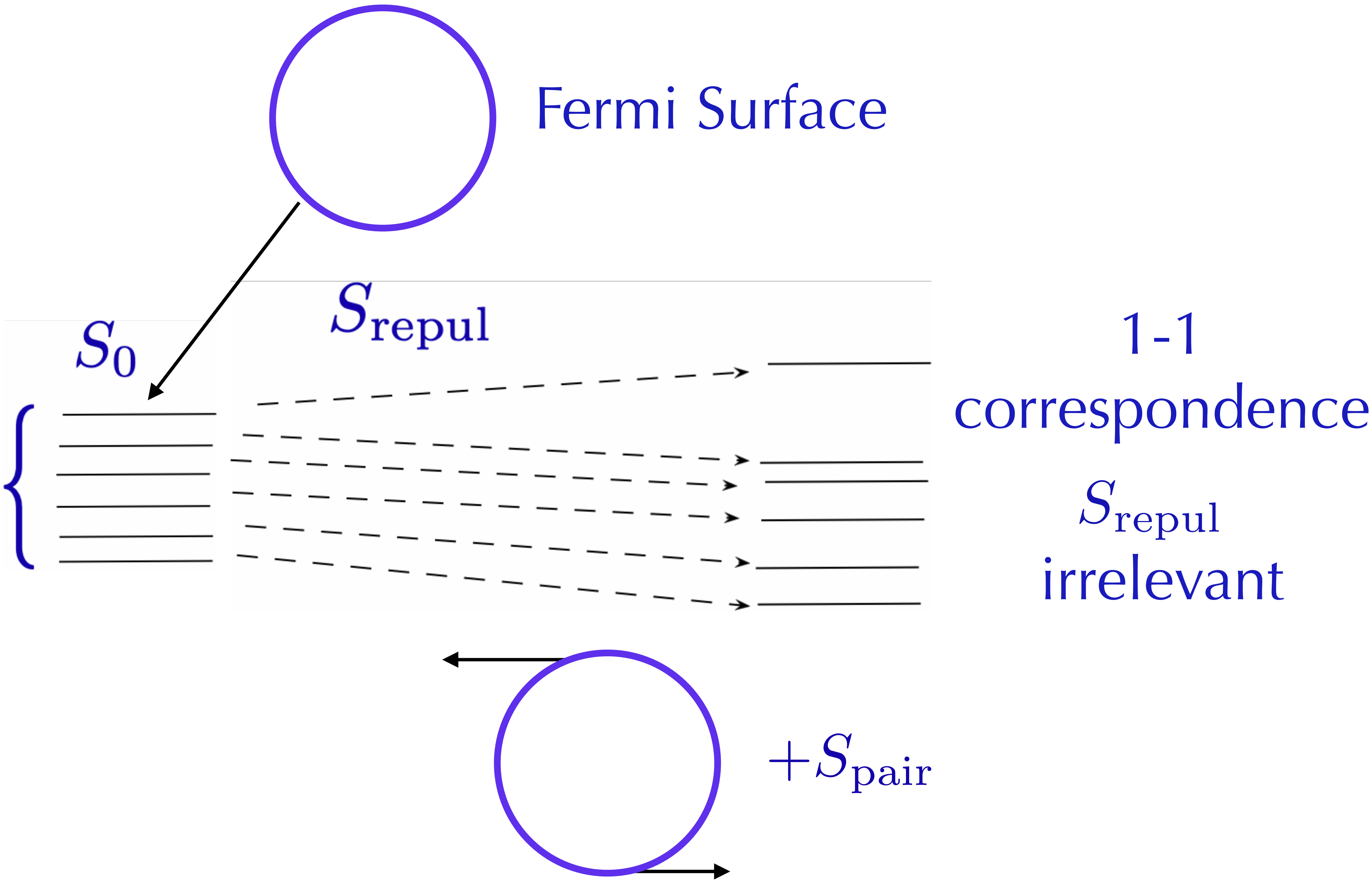


$-g b_p^\dagger b_{p'}$

$E_b \propto e^{1/g}$

$\langle \psi^\dagger \psi^\dagger \rangle = \Delta e^{i\phi}$

$$\mathrm{FL} \to \mathrm{BCS}$$



superconductivity $\dfrac{2\Delta}{T_c} = 3.5$

Fermi gas

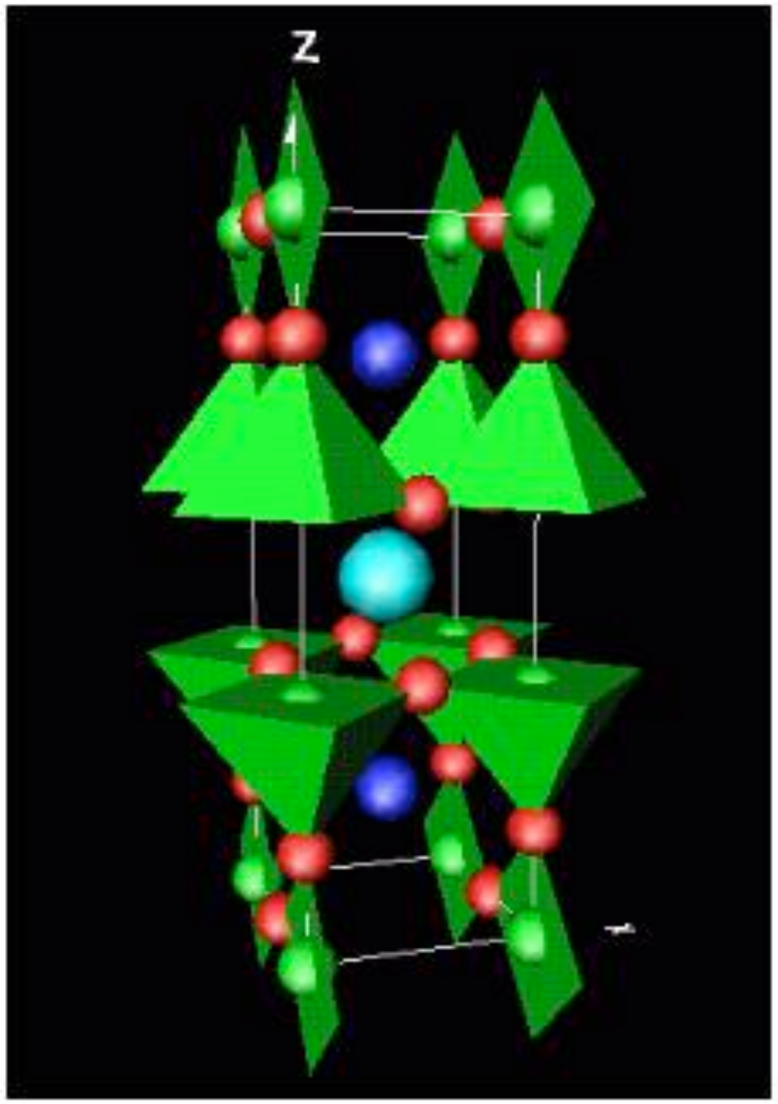
Fermi liquid

BCS superconductor

?

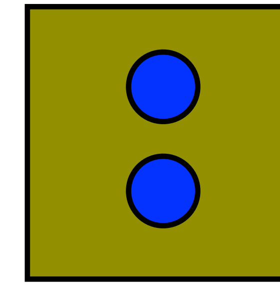
Is there physics
beyond
BCS?

fixed
point beyond
FL?

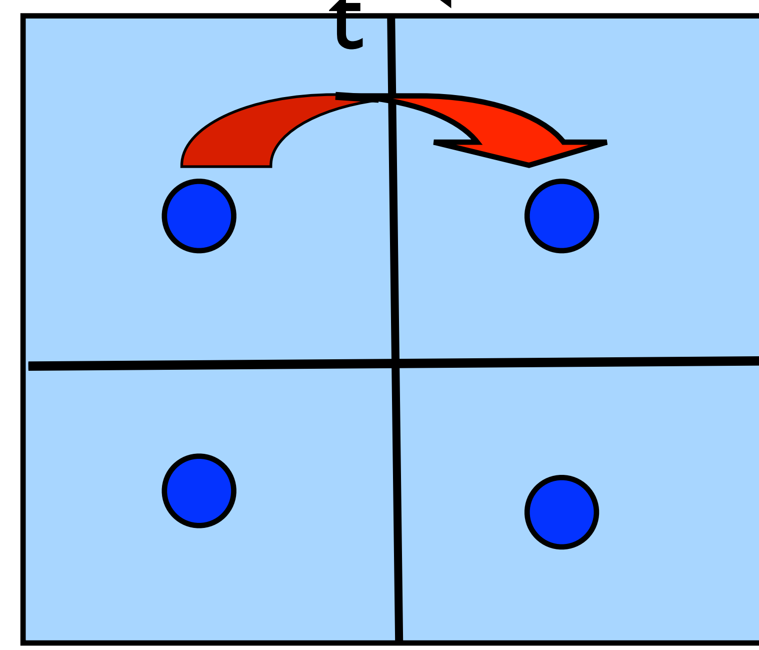


$Y Ba_2 Cu_3 O_7$
Cuprate Superconductors

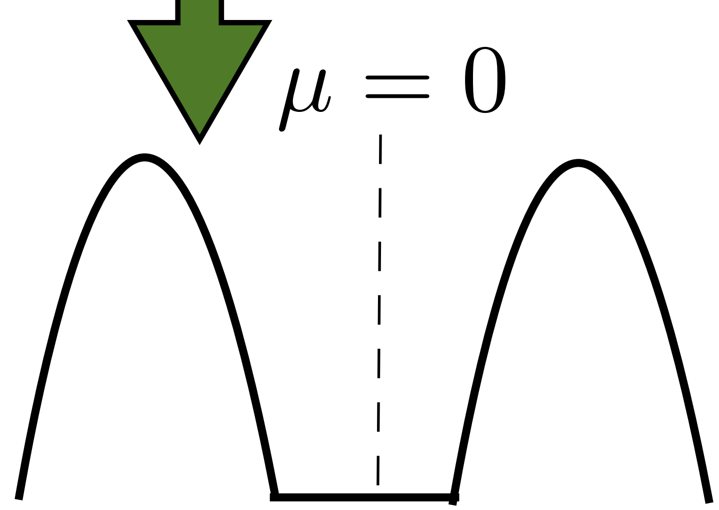
NiO insulates
d^8 ?
perhaps this costs energy



$U \gg t$



local real-space physics



no change in size of Brillouin zone

solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT

QMC

disputes

Sept. 1997

A Critique of Two Metals

R. B. Laughlin
Departrment of Physics
Stanford University
Stanford, California 94305

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997

A Critique of "A Critique of Two Metals"

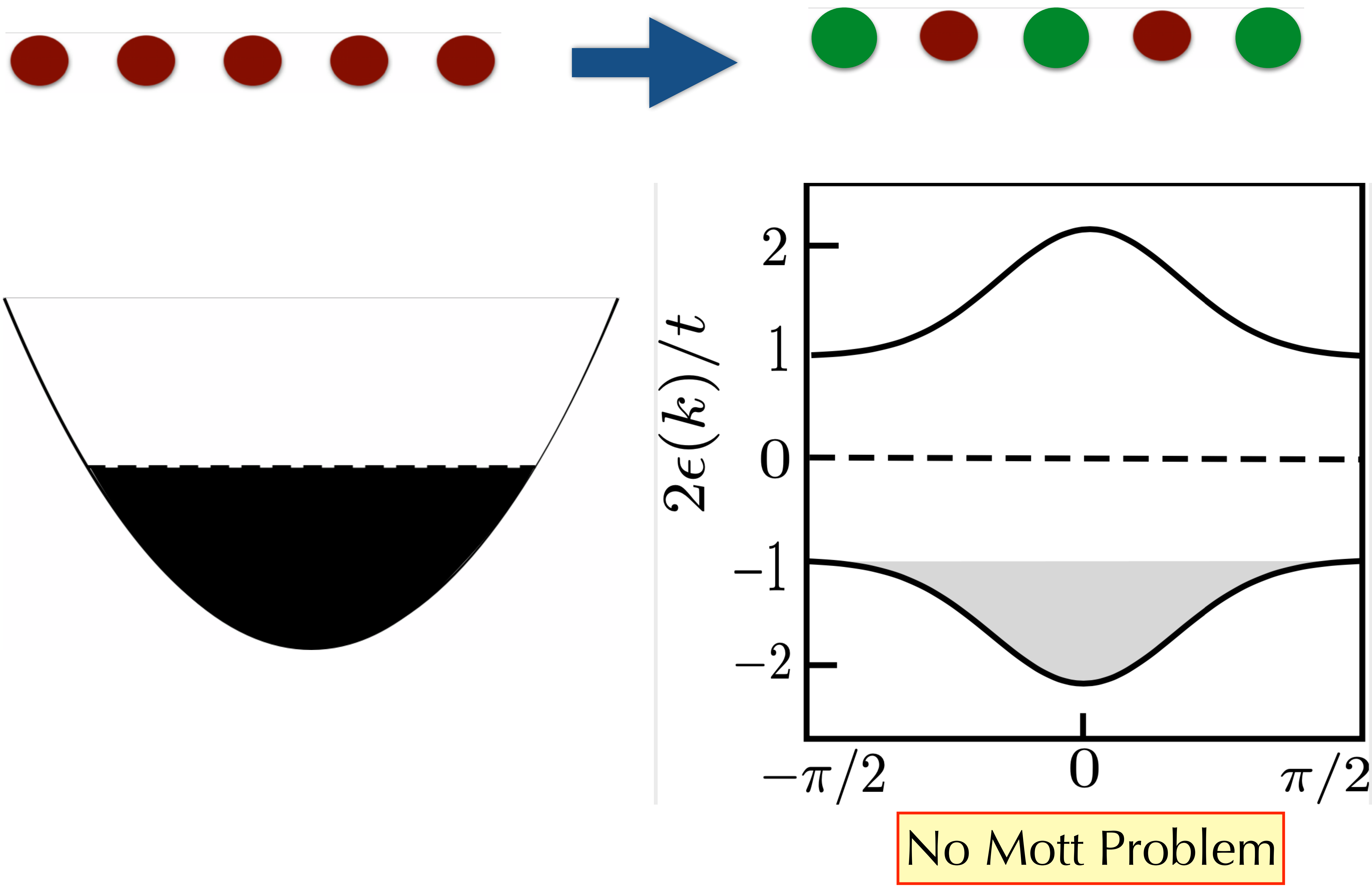
Philip W. Anderson and G. Baskaran

Joseph Henry Laboratories of Physics

Princeton University, Princeton, NJ 08544

The fundamental argument is presented in the second paragraph: "Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration"...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso "at zero temperature" is added, because of course most Mott

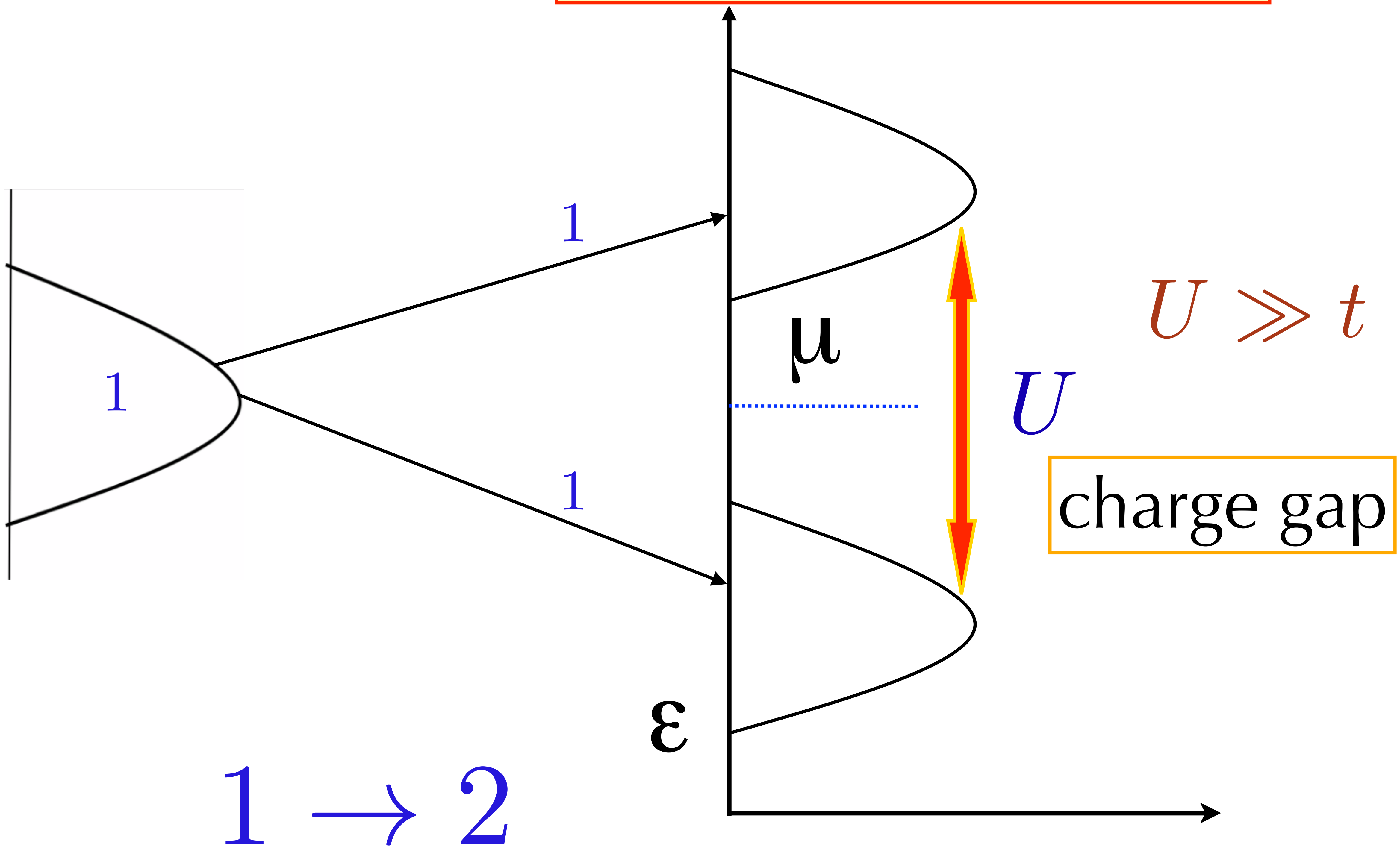
concern. It is the tragedy of Mott that although he almost certainly won his Nobel prize for the Mott insulator, Slater, who couldn't think clearly about finite temperature, won the publicity battle.



2
1
0
−1
−2
2ε(k)/t
−π/2
0
π/2
No Mott Problem

Laughlin's objection:

gap with no symmetry breaking not demonstrated!!



$-$

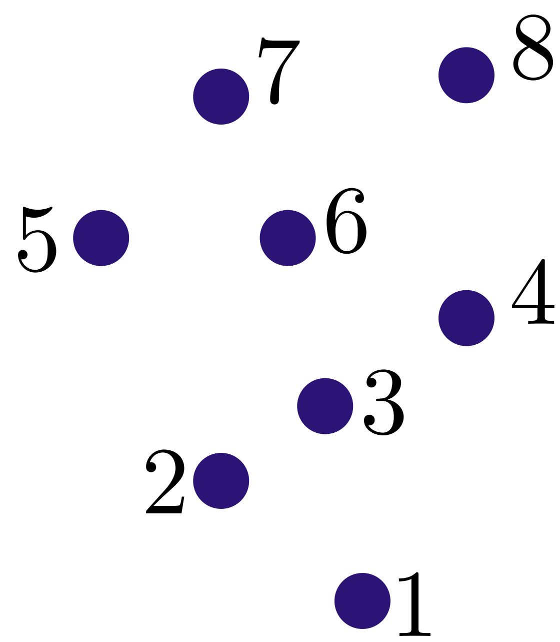
$=$ Mottness

zeros

$= 0$ $\neq 0$

$\mathrm{DetReG}(\omega = 0, \mathrm{p})$

counting particles

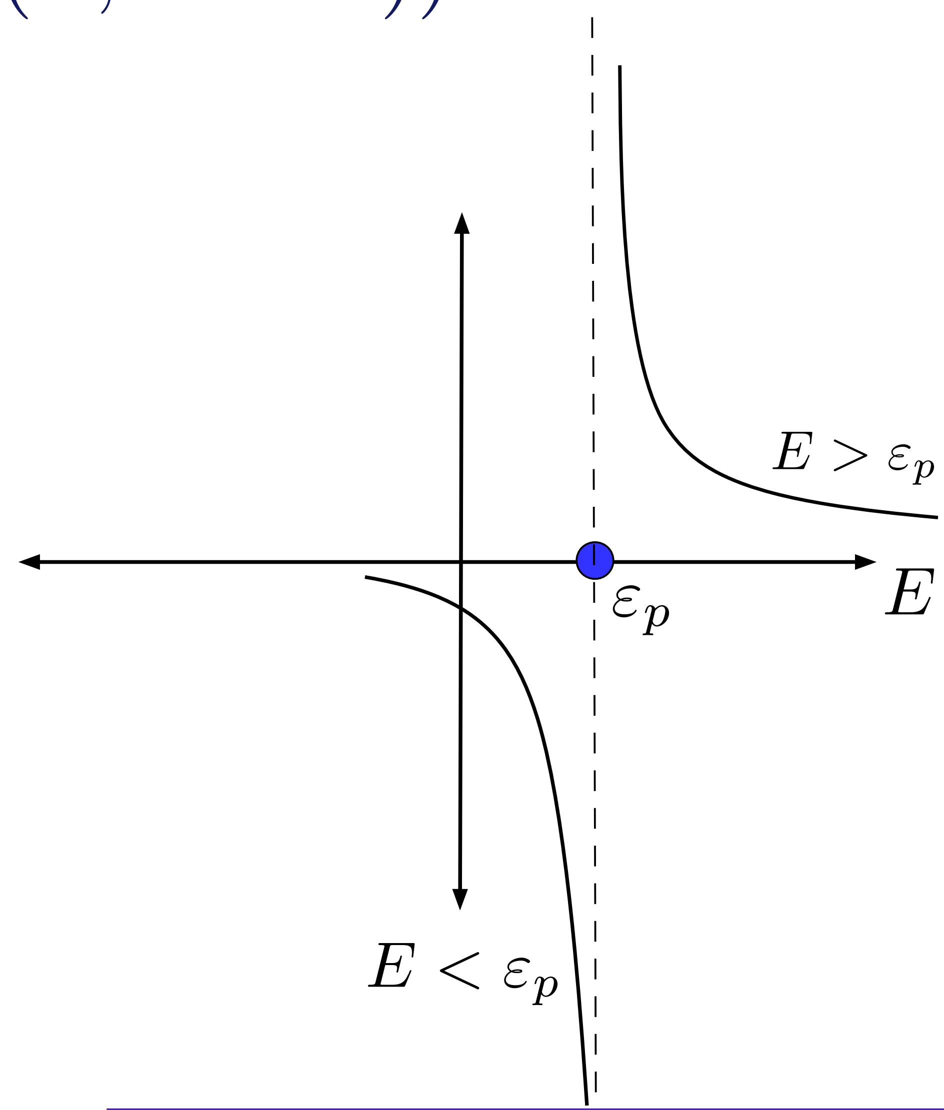


is there a more efficient way?

Luttinger counting theorem

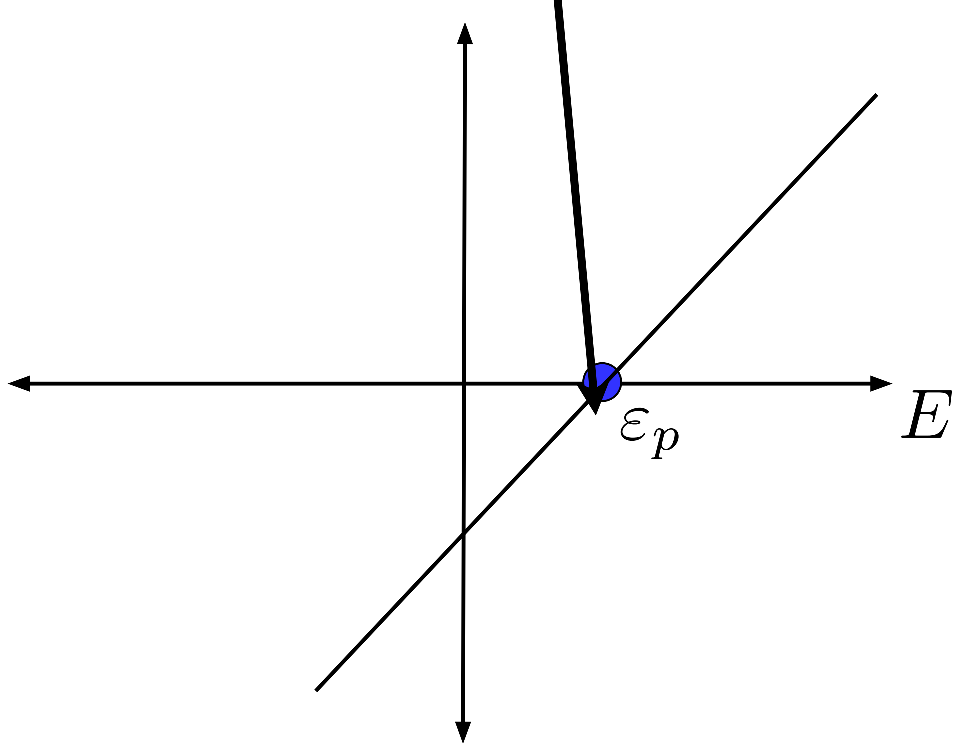
$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2\sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$



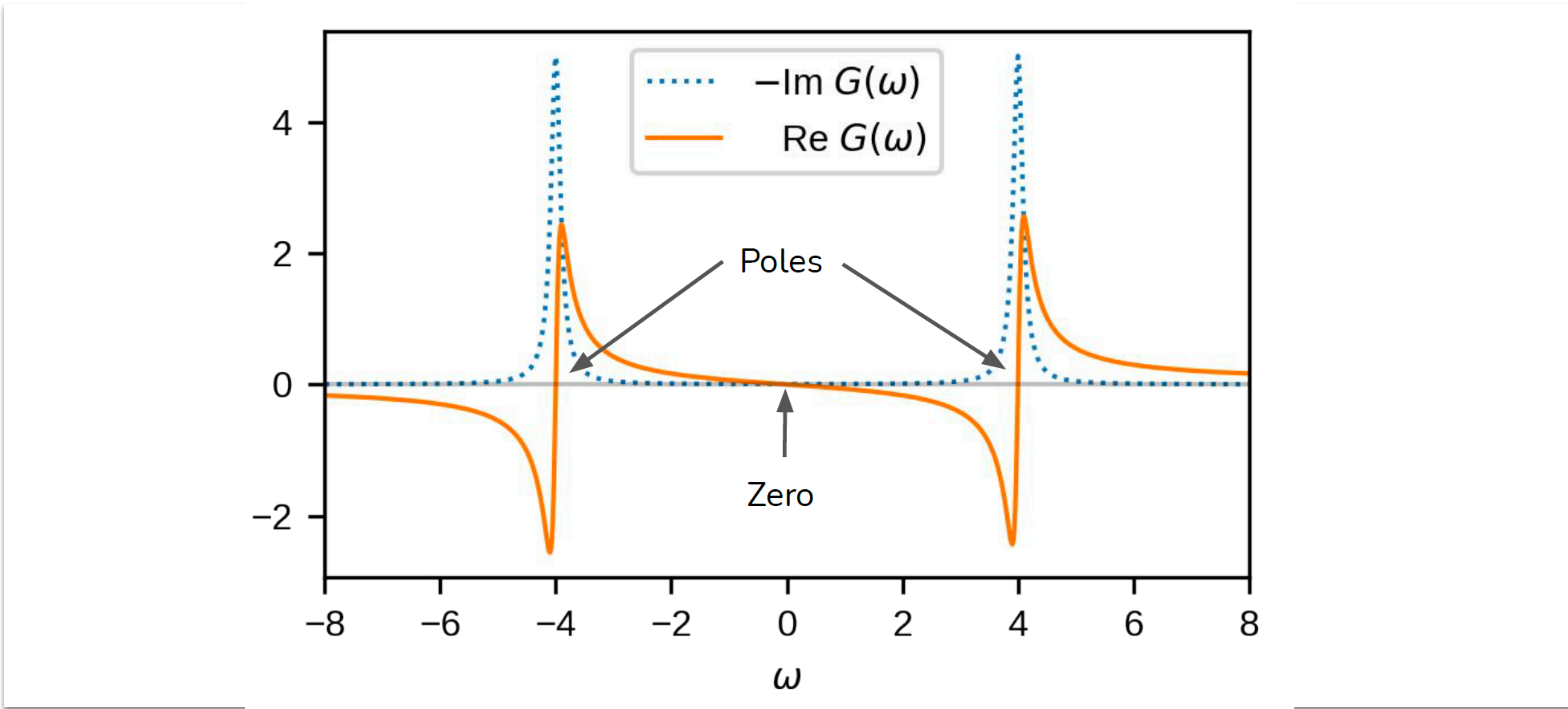
zero-crossing

$$\mathrm{Det}G(\omega = 0, \vec{p}) = 0$$



counting poles (qp)

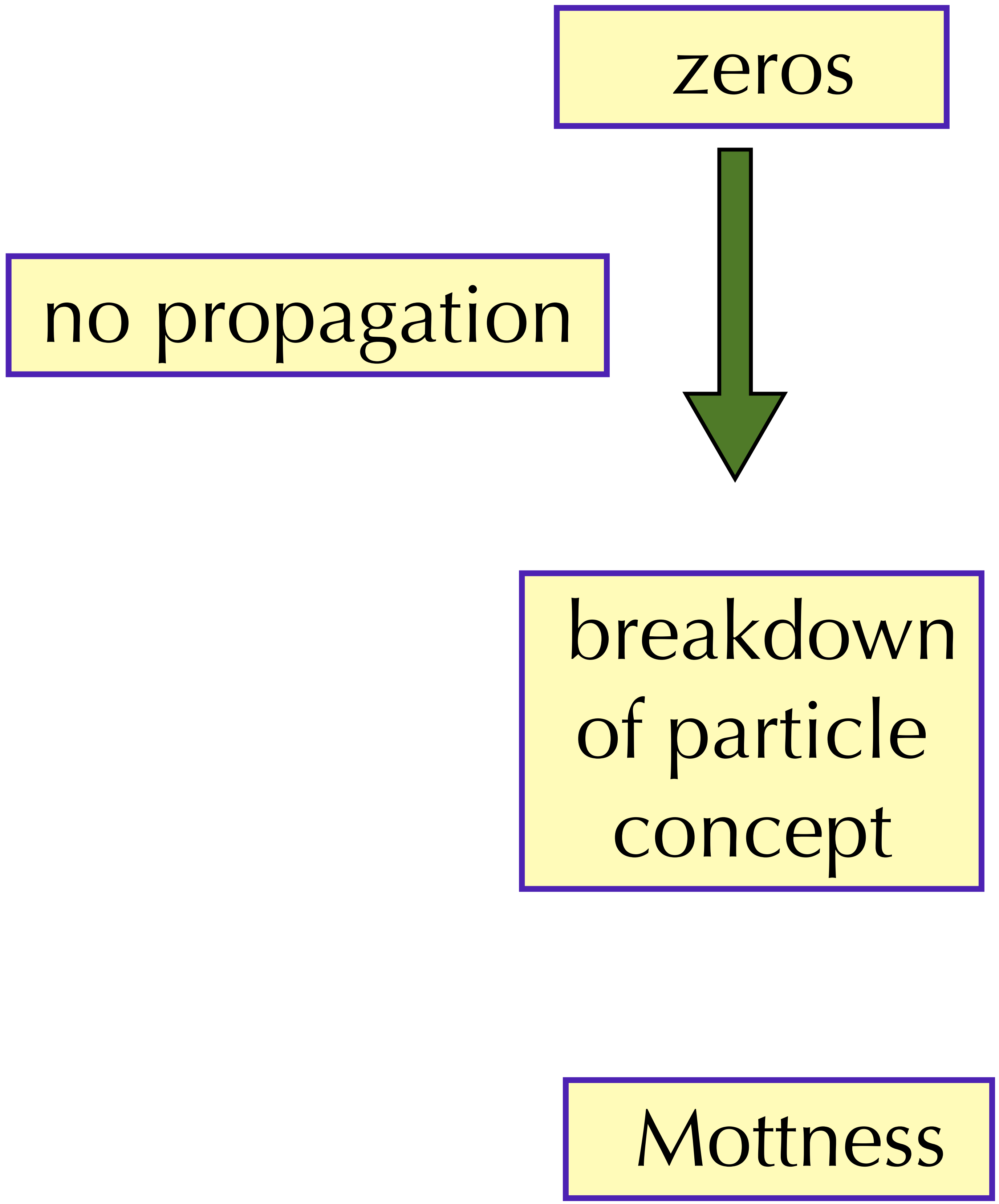
How do zeros obtain?



= below gap+above gap = 0

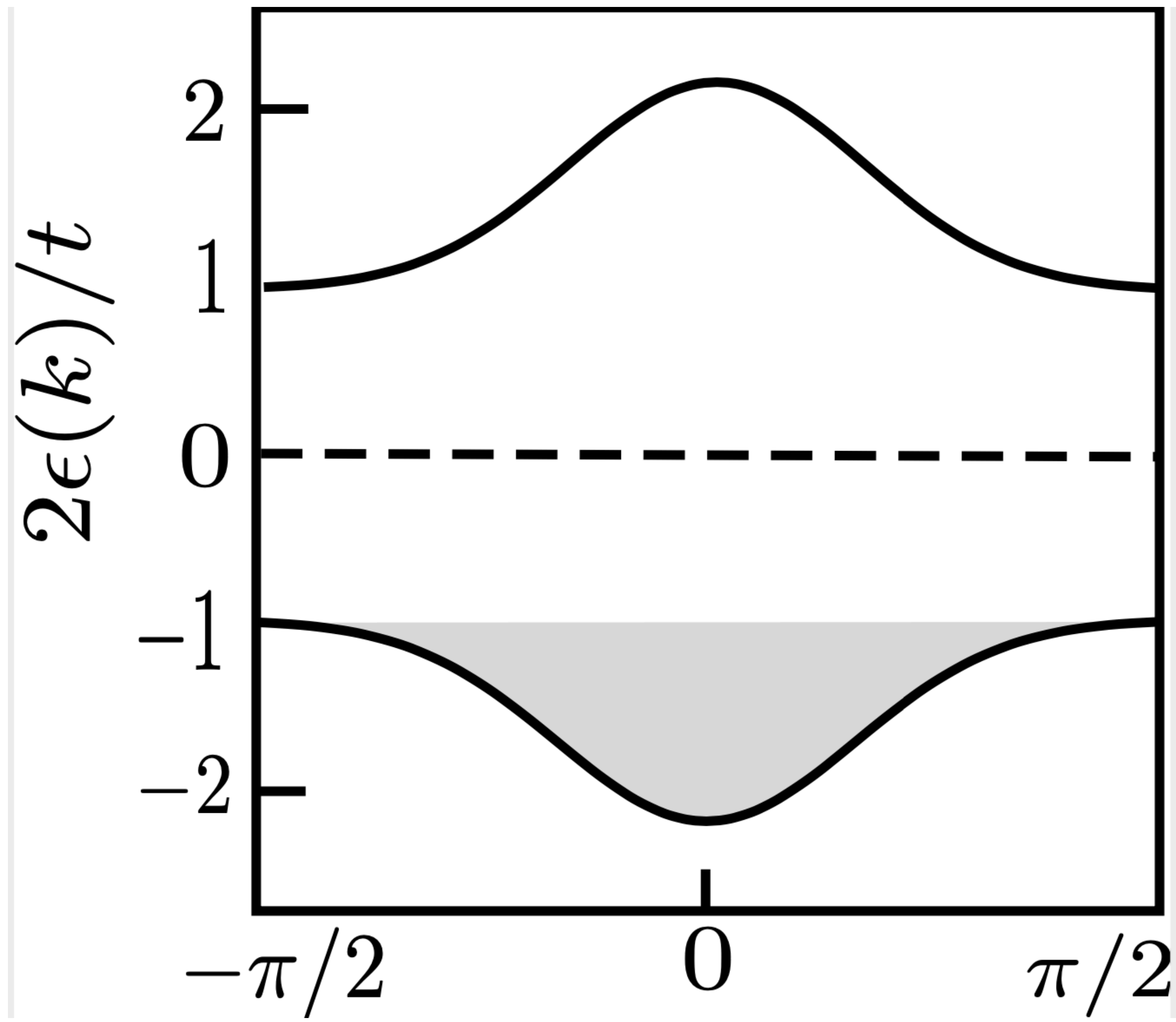
$$\mathrm{DetReG}(k, \omega = 0) = 0$$ (single band)

strongly correlated gapped systems


zeros
no propagation
breakdown of particle concept
Mottness

Symmetry Breaking

$$G_k(\omega) = \begin{pmatrix} \frac{1}{\omega - E_k^+} & 0 \\ 0 & \frac{1}{\omega - E_k^-} \end{pmatrix}$$

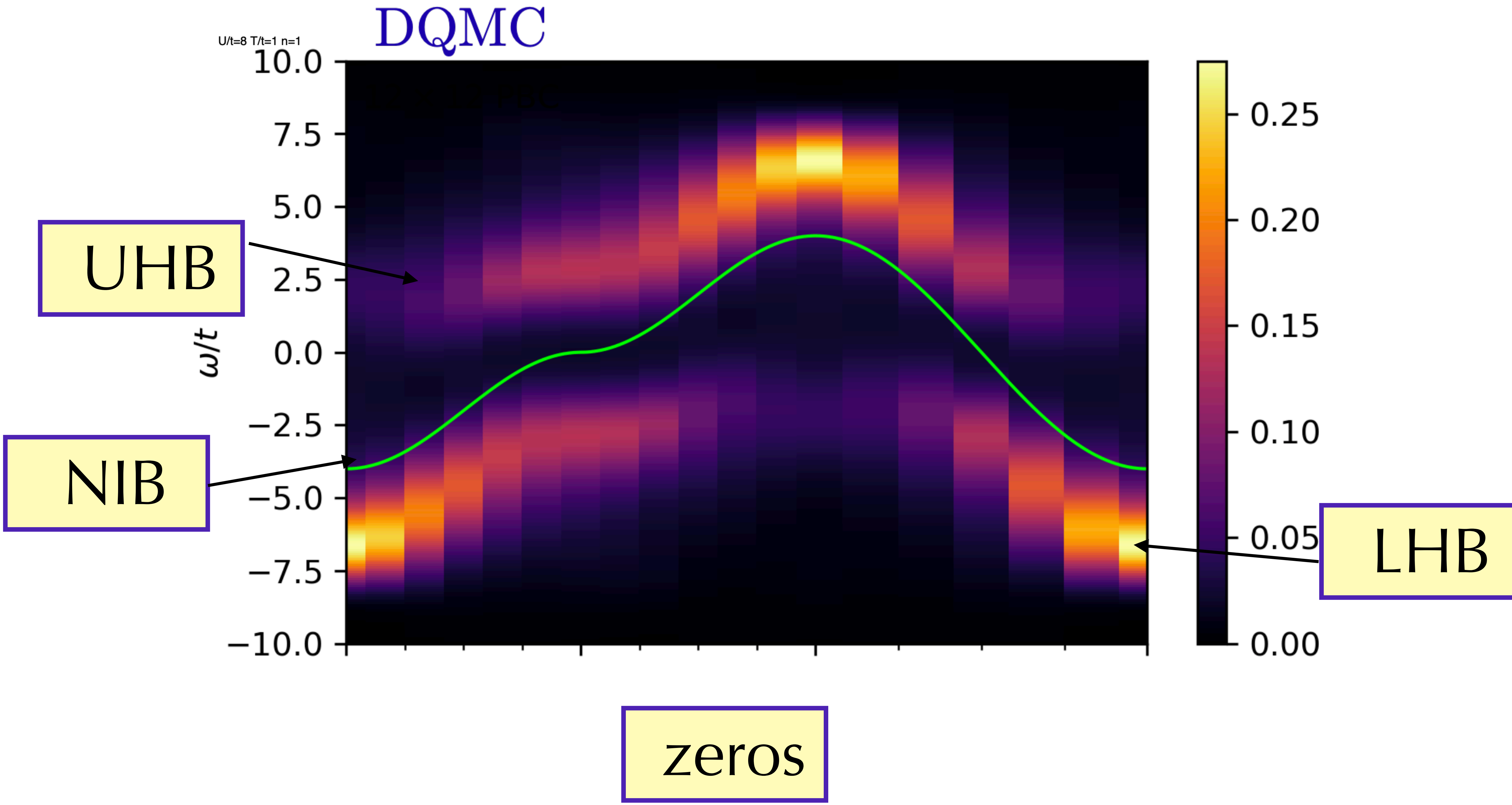


$$\mathrm{Det}G \neq 0$$

no Mottness

Laughlin

Is the Hubbard model necessary?



No!

Minimal model for Mottness?

Anderson
Haldane
2000

2 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + \cdots \quad (\cdots \to 0)$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$\to (c_{p\uparrow}, c_{p\downarrow}, \mathrm{h.c.})$ 4 objects

$O(4)$

$\mathrm{Det M} = 1$ → proper rotations SO(4)

$\mathrm{Det M} = -1$ → improper rotations

$$\mathrm{Det}\, M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

$$\epsilon(p) = \epsilon_F$$

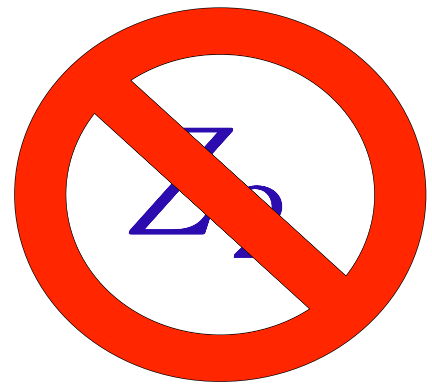
Fermi Surface

$$H = 0$$

$$\left.\begin{array}{l} n_{p\uparrow} \to -n_{p\uparrow} \\ n_{p\downarrow} \to n_{p\downarrow} \end{array}\right\} Z_2$$

at Fermi surface only

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F)\, n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd under Z_2

scaling dimension

$[n_{p\uparrow} n_{p\downarrow}] = -2$

New fixed point!

relevant interaction

Hatsugai-Kohmoto model

Hubbard not necessary!

Hatsugai-Kohmoto Model (1992)

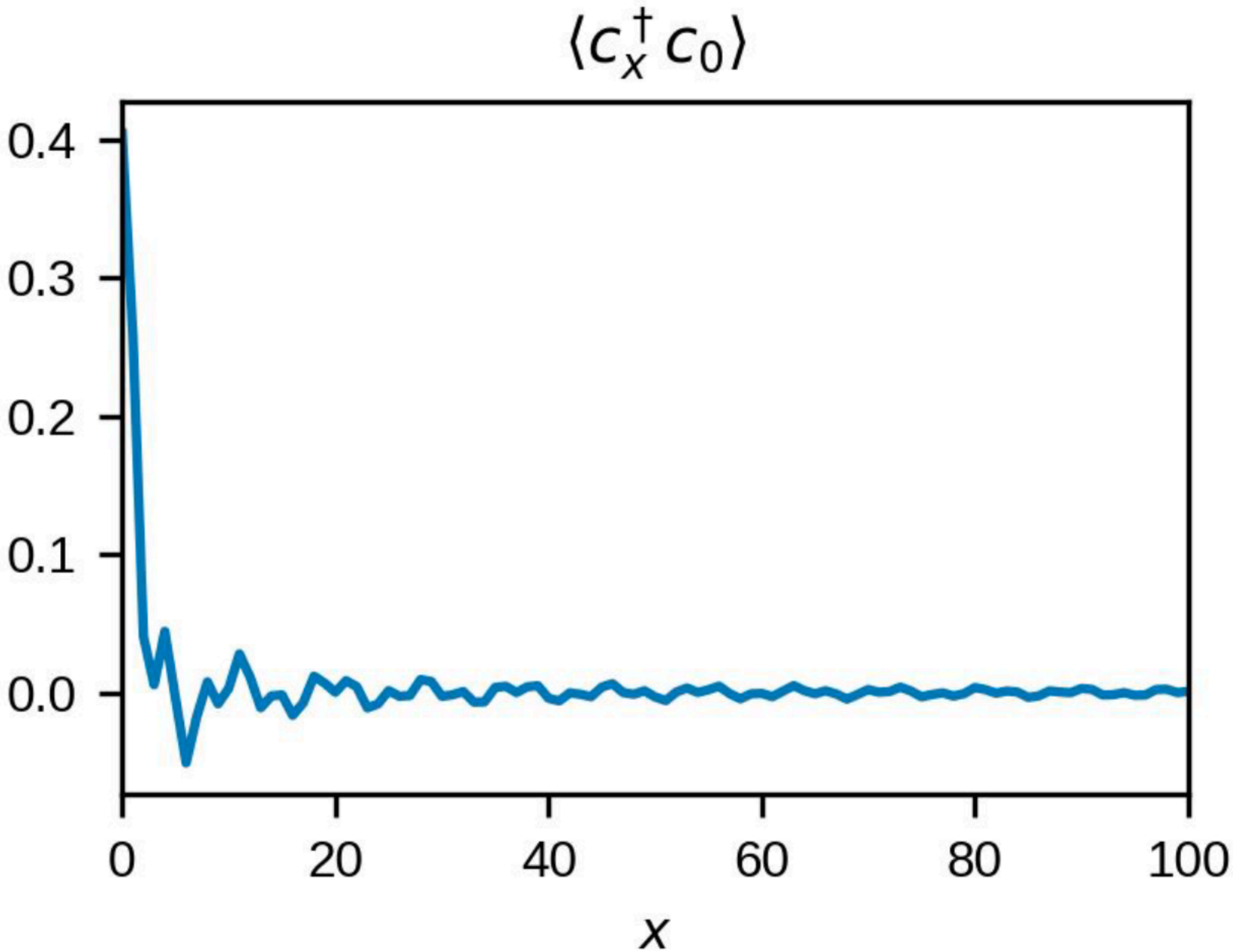
$$H_{\rm HK} = \qquad -t\sum_{\langle j,l\rangle,\sigma}\left(c^\dagger_{j\sigma}c_{l\sigma}+h.c.\right)-\mu\sum_{j\sigma}c^\dagger_{j\sigma}c_{j\sigma}$$

$$c_{k\sigma}=\sum_j e^{ikj}c_{j\sigma}$$

$$H_{\rm HK}=\sum_k H_k=\sum_k\left(\xi_k(n_{k\uparrow}+n_{k\downarrow})+Un_{k\uparrow}n_{k\downarrow}\right).$$

$$\xi_k=\epsilon_k-\mu$$

non-local but correlations are local



General HK Model

$$\sum_k \left(\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k'\downarrow}\right).$$

relevant perturbation

$$[H_t, H_U] = 0$$

Solvable Mott transition

$$G_{k\sigma}(i\omega_n \to z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)}$$

lower Hubbard band

upper Hubbard band

Hubbard band operators

$$c^\dagger_{k\sigma}$$

$$\zeta^\dagger_{k\sigma} = c^\dagger_{k\sigma}(1 - n_{k\bar{\sigma}})$$

$$c^\dagger_{k\sigma} n_{k\bar{\sigma}} = \eta^\dagger_{k\sigma}$$

$+$

$$\langle n_{k\sigma} \rangle = \frac{1}{2}$$

$$G^R_\sigma(k,\omega) = \frac{1}{\omega + i0^+ - (\xi_k + U/2) - \frac{(U/2)^2}{\omega + i0^+ - (\xi_k + U/2)}}$$

$$\Sigma(k,\omega)$$

$$\omega = \xi_k + U/2$$

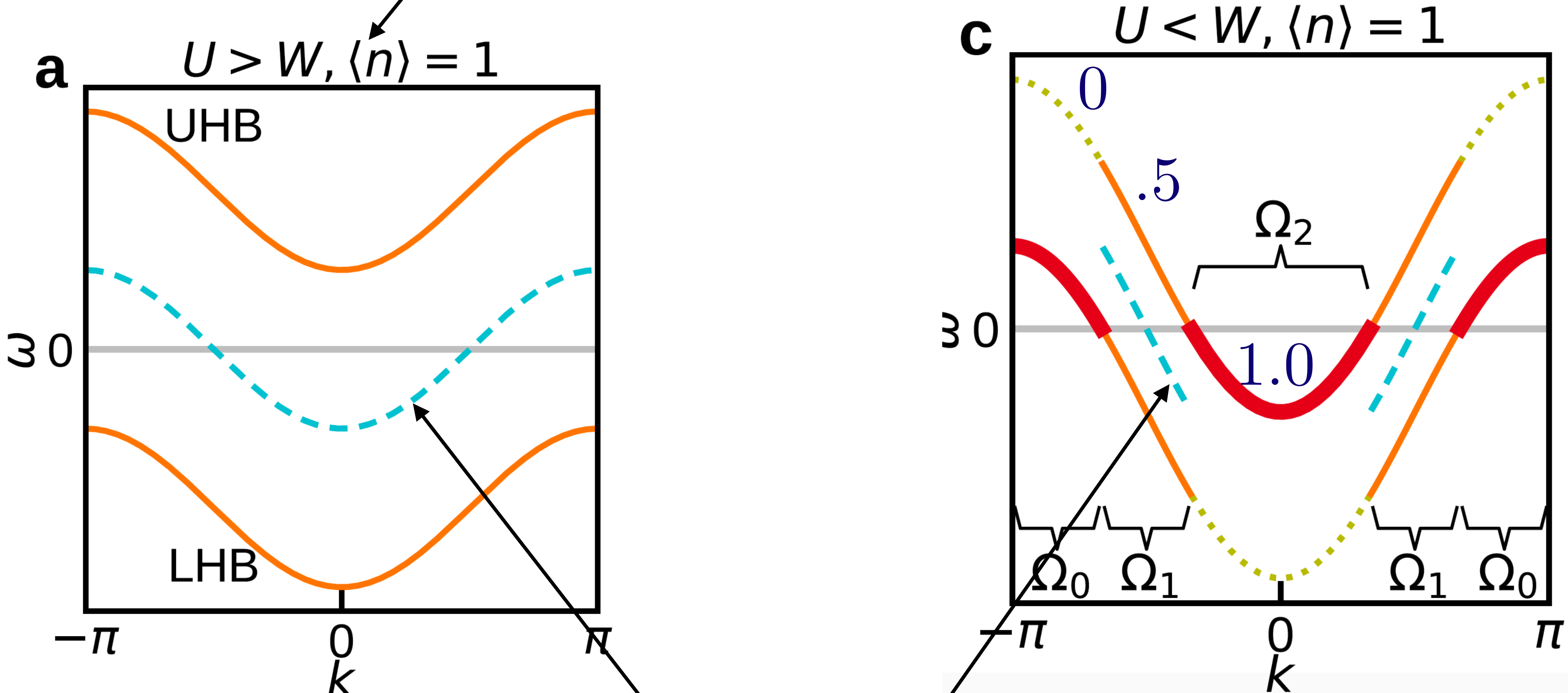
$$\Re\Sigma = \Im\Sigma = \infty$$

Fermi liquid QP

zeros

Mott transition: composite excitations

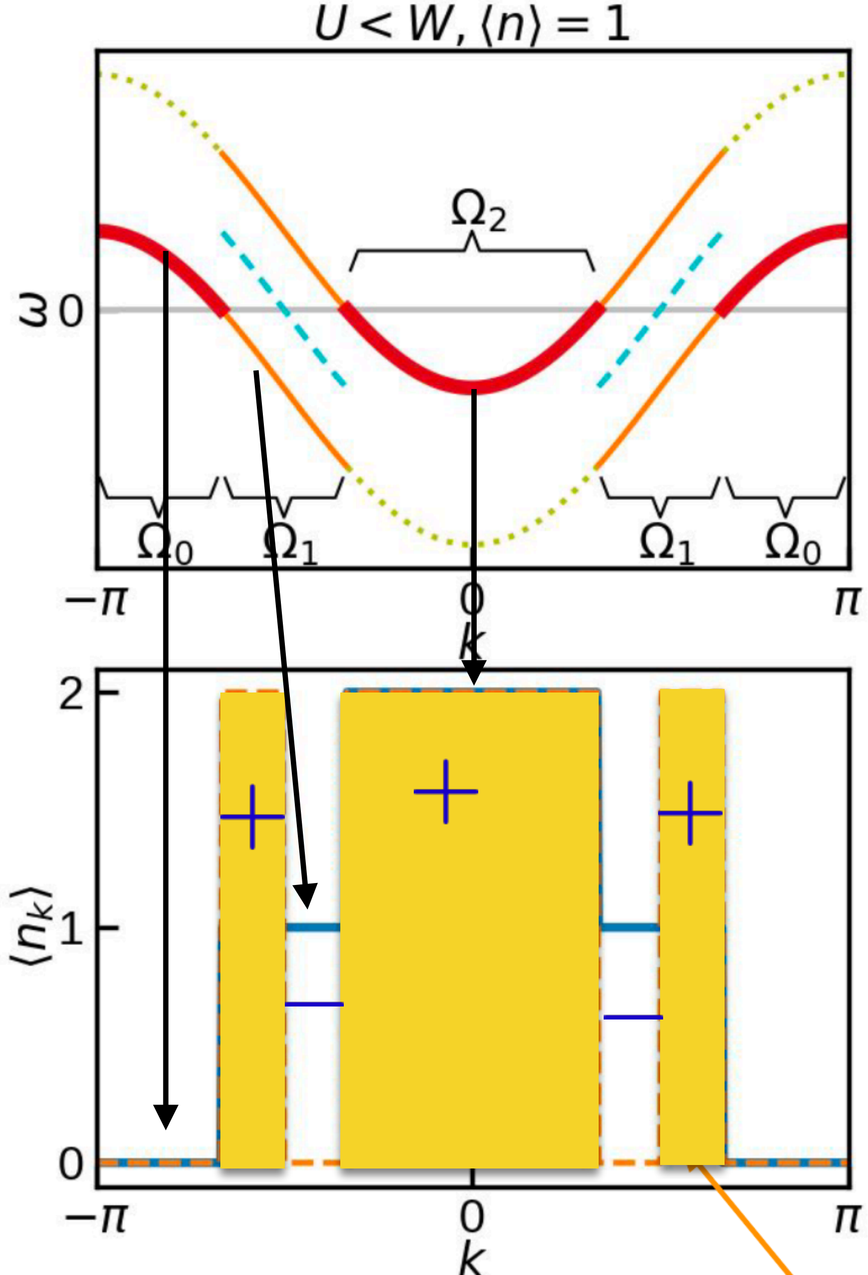
$$\Delta E = U - 4dt = U - W$$



insulator

metal

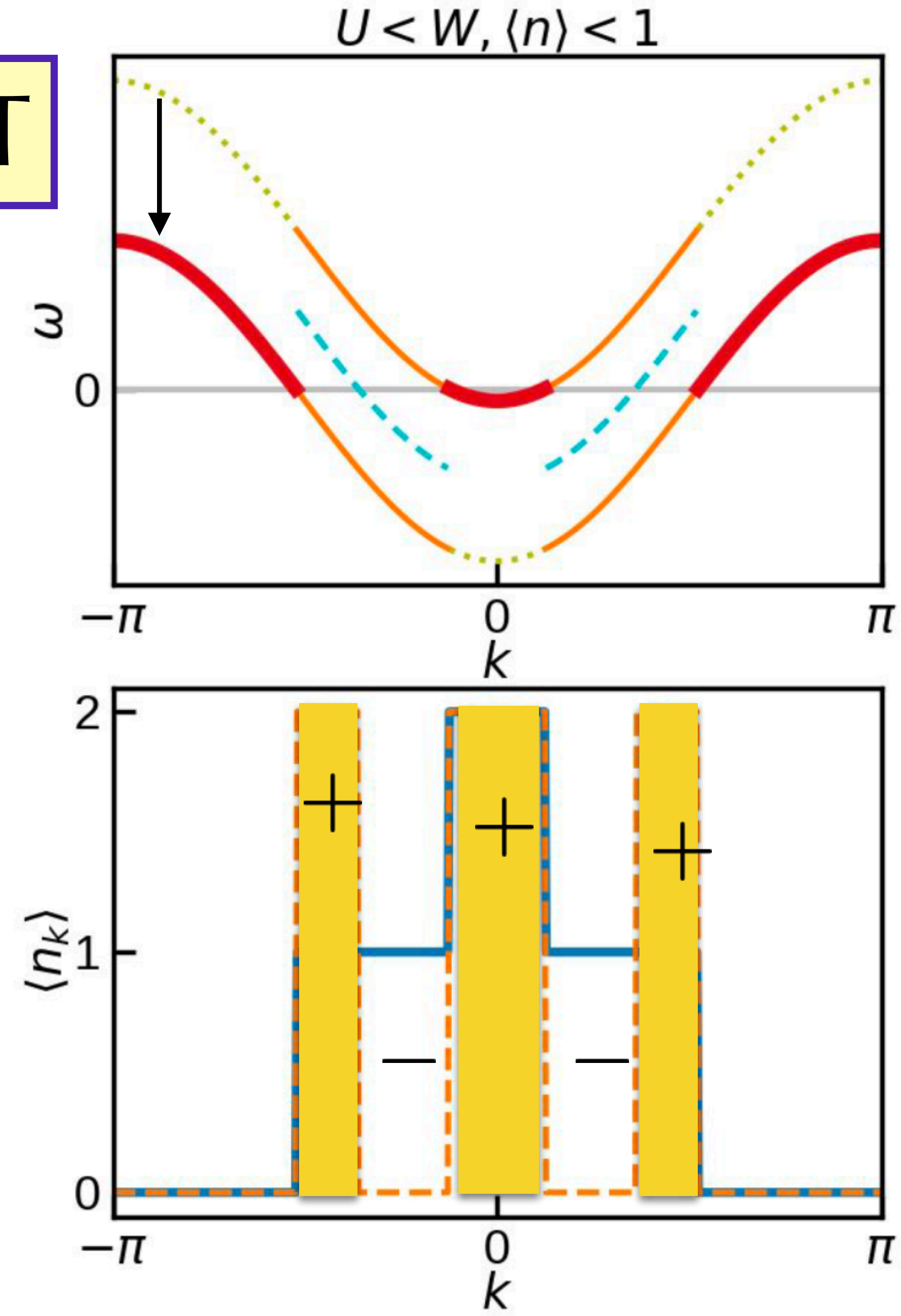
Luttinger Surface



$n_{\mathrm{Lutt}} = \langle n \rangle$

$2\theta(\mathrm{Re}\, G(\mathbf{k}, \omega = 0))$

SWT

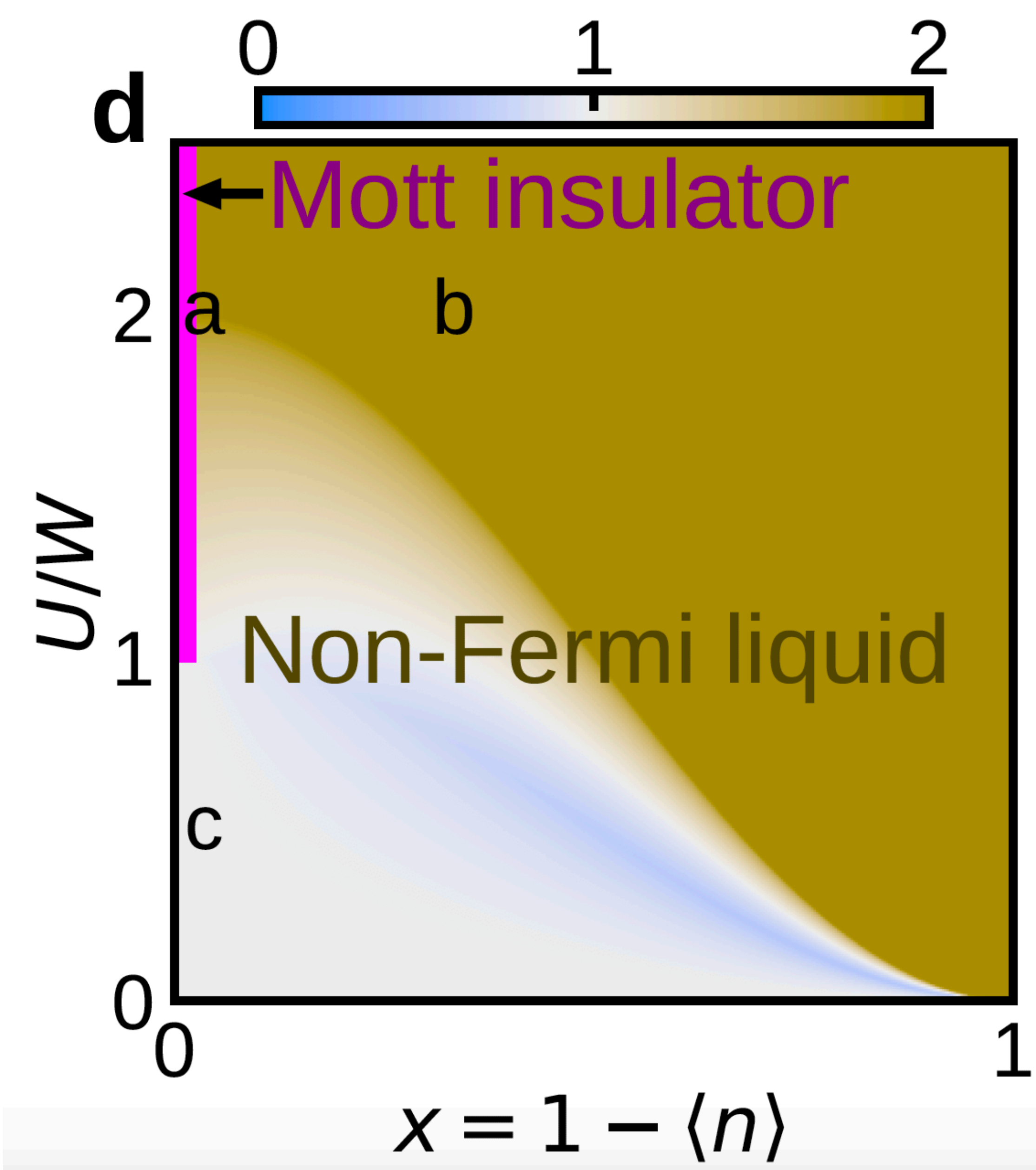


$n_{\mathrm{Lutt}} \neq \langle n \rangle$

Why NFL?

FL

HK Mottness



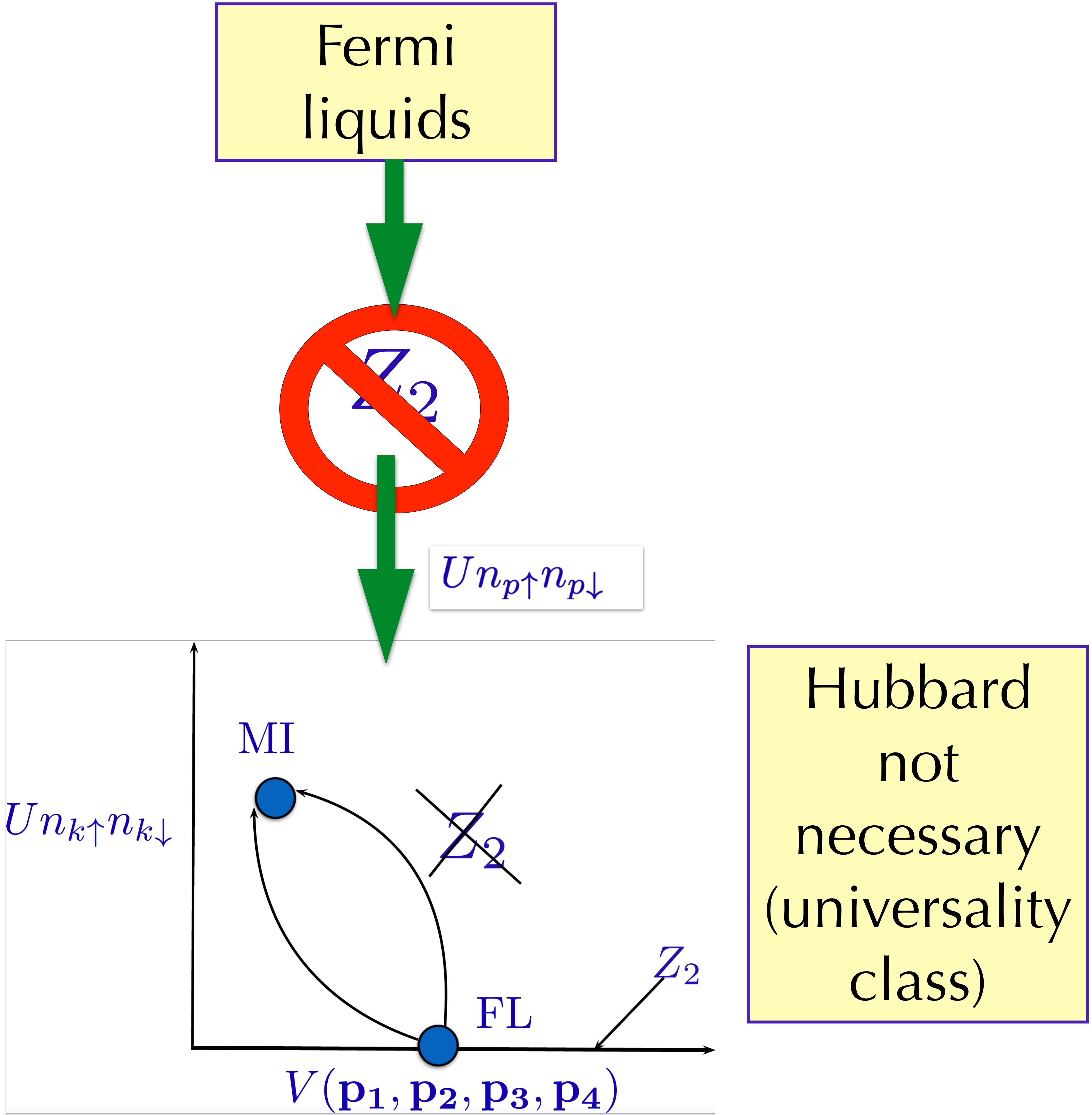
$$c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|G\rangle \not\leftrightarrow \ ?$$

$$c^\dagger_{k\uparrow}|G\rangle, c^\dagger_{k\downarrow}|G\rangle \leftrightarrow \zeta^\dagger_{k\uparrow}|G\rangle, \zeta^\dagger_{k\downarrow}|G\rangle$$

$$|G\rangle \qquad |G\rangle$$

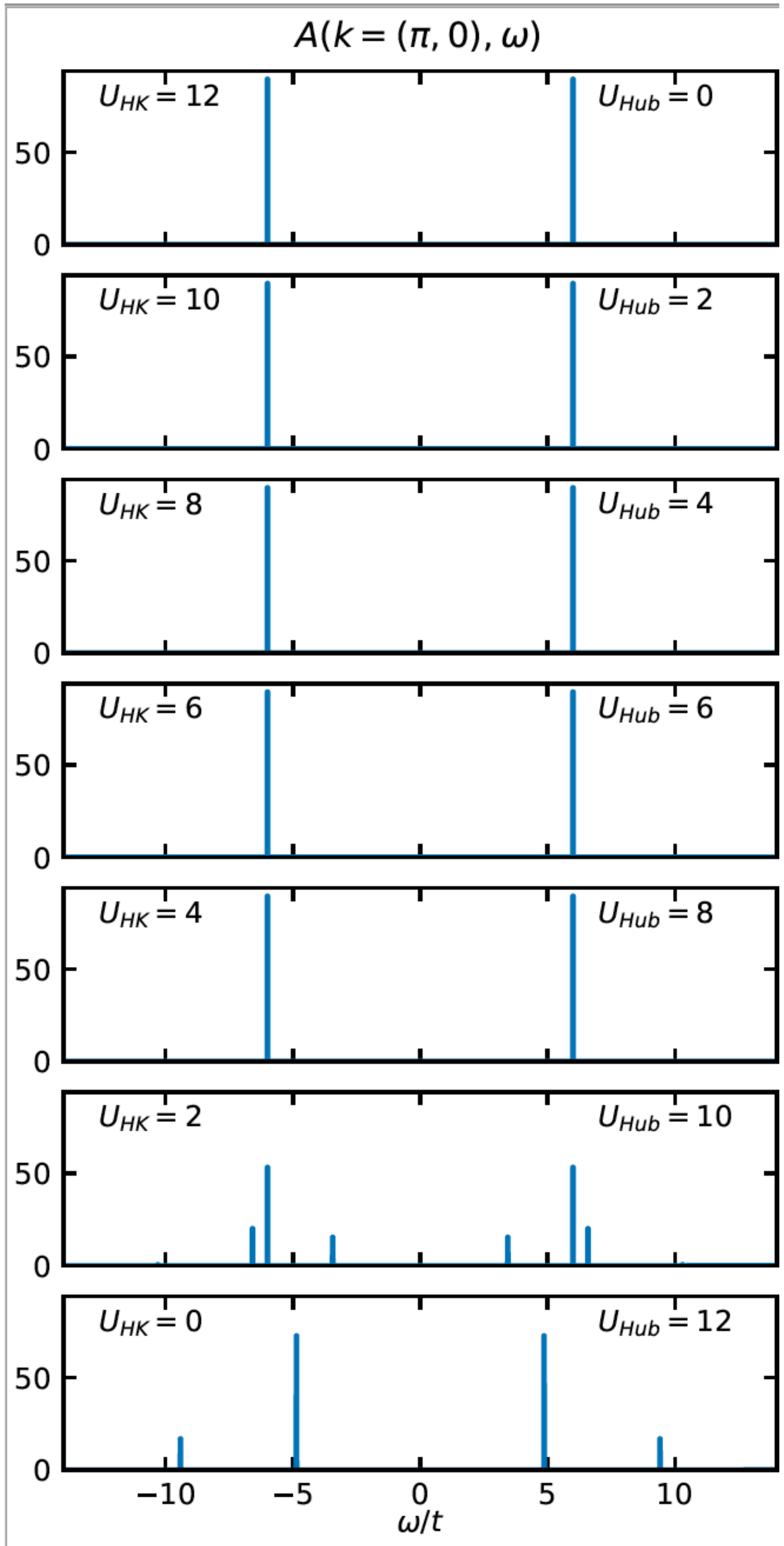
$$\zeta^\dagger_{k\uparrow}\zeta^\dagger_{k\downarrow}|G\rangle = 0$$

EFL?


Fermi liquids
Z_2
$U n_{p\uparrow} n_{p\downarrow}$
MI
$U n_{k\uparrow} n_{k\downarrow}$
Z_2
Z_2
FL
$V(\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}, \mathbf{p_4})$
Hubbard not necessary (universality class)

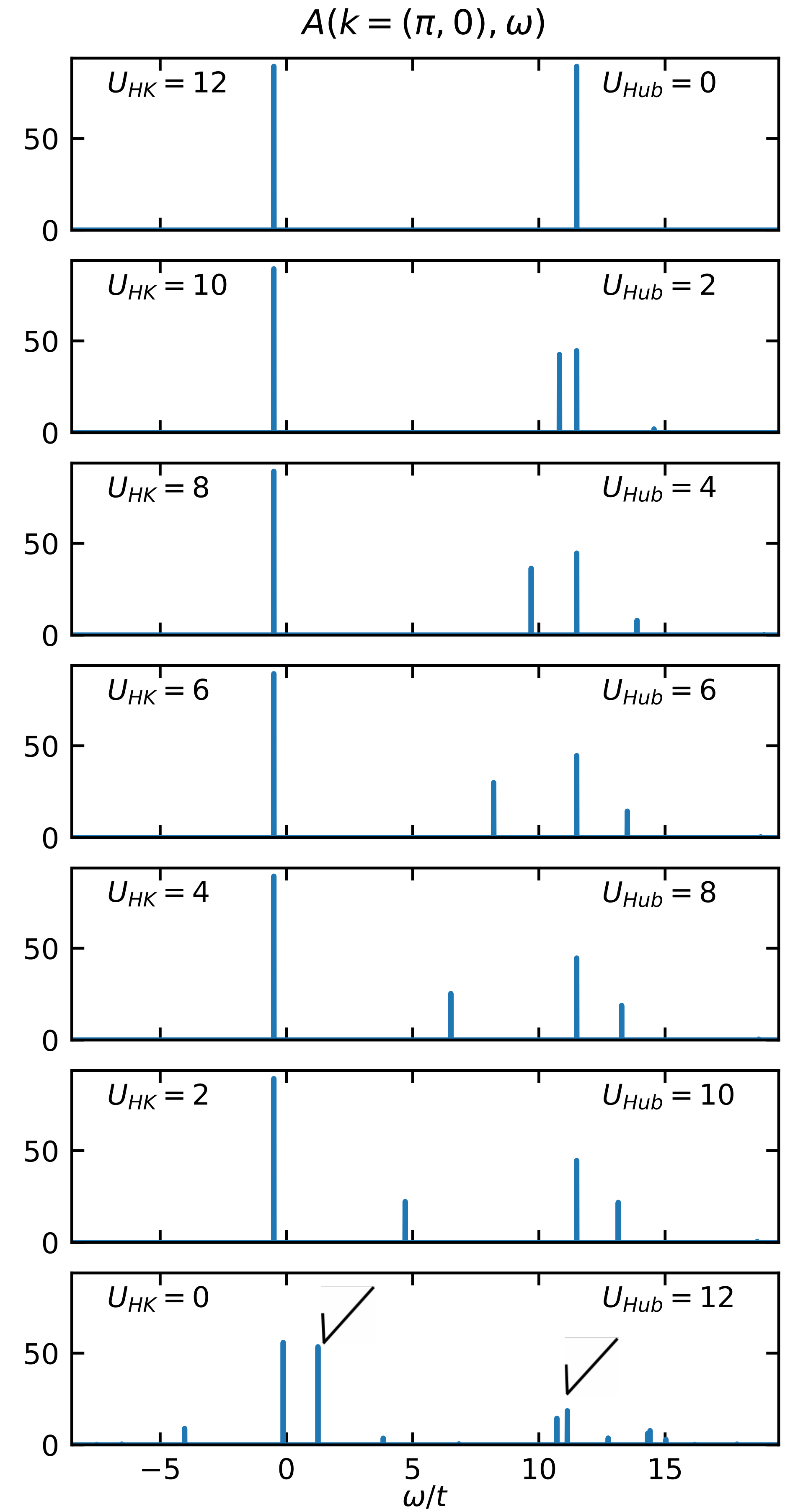
$n = 1.0$

$H_{\mathrm{HK}} \approx H_{\mathrm{Hubb}}$



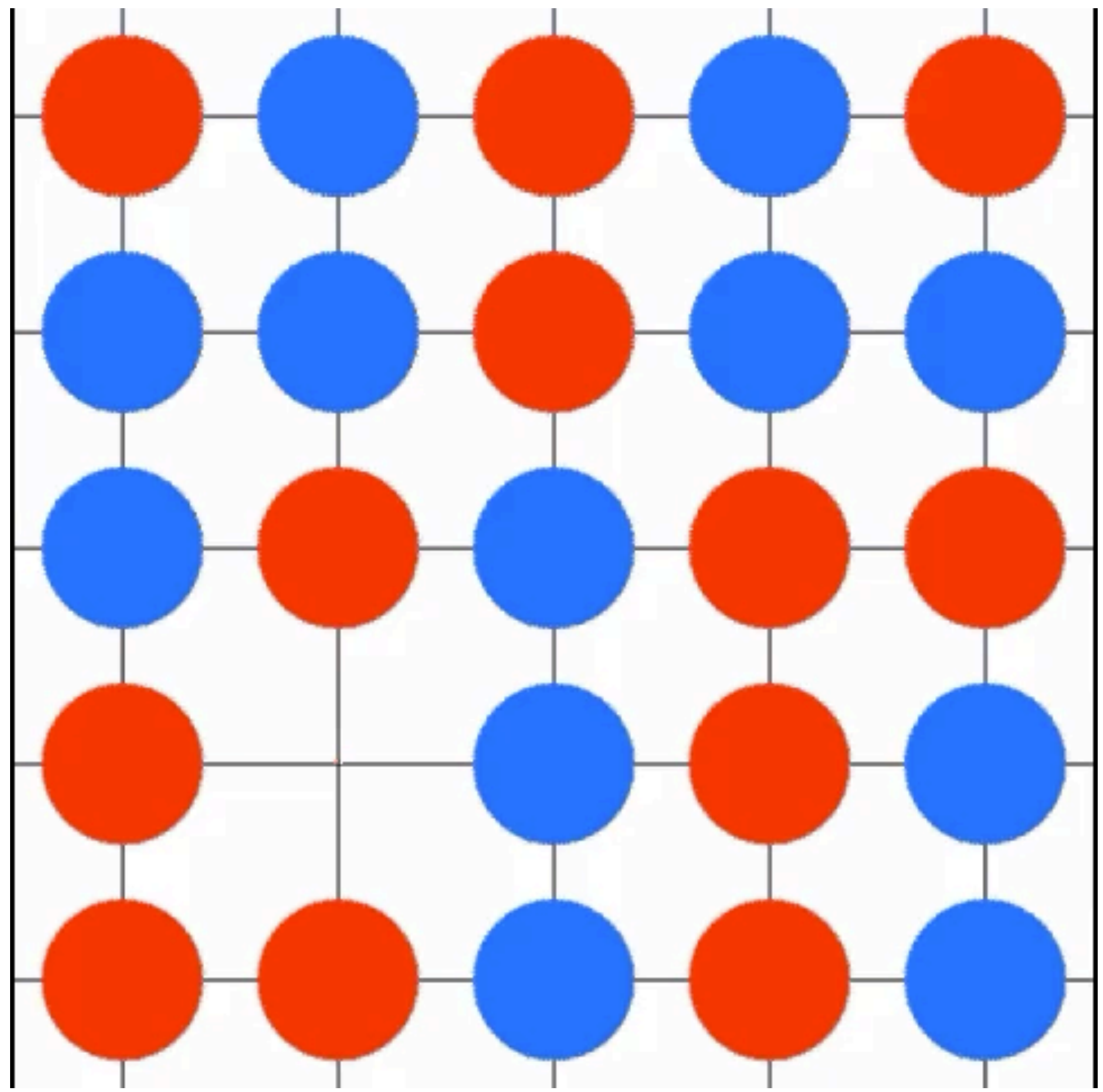
$n = 0.875$

DSWT

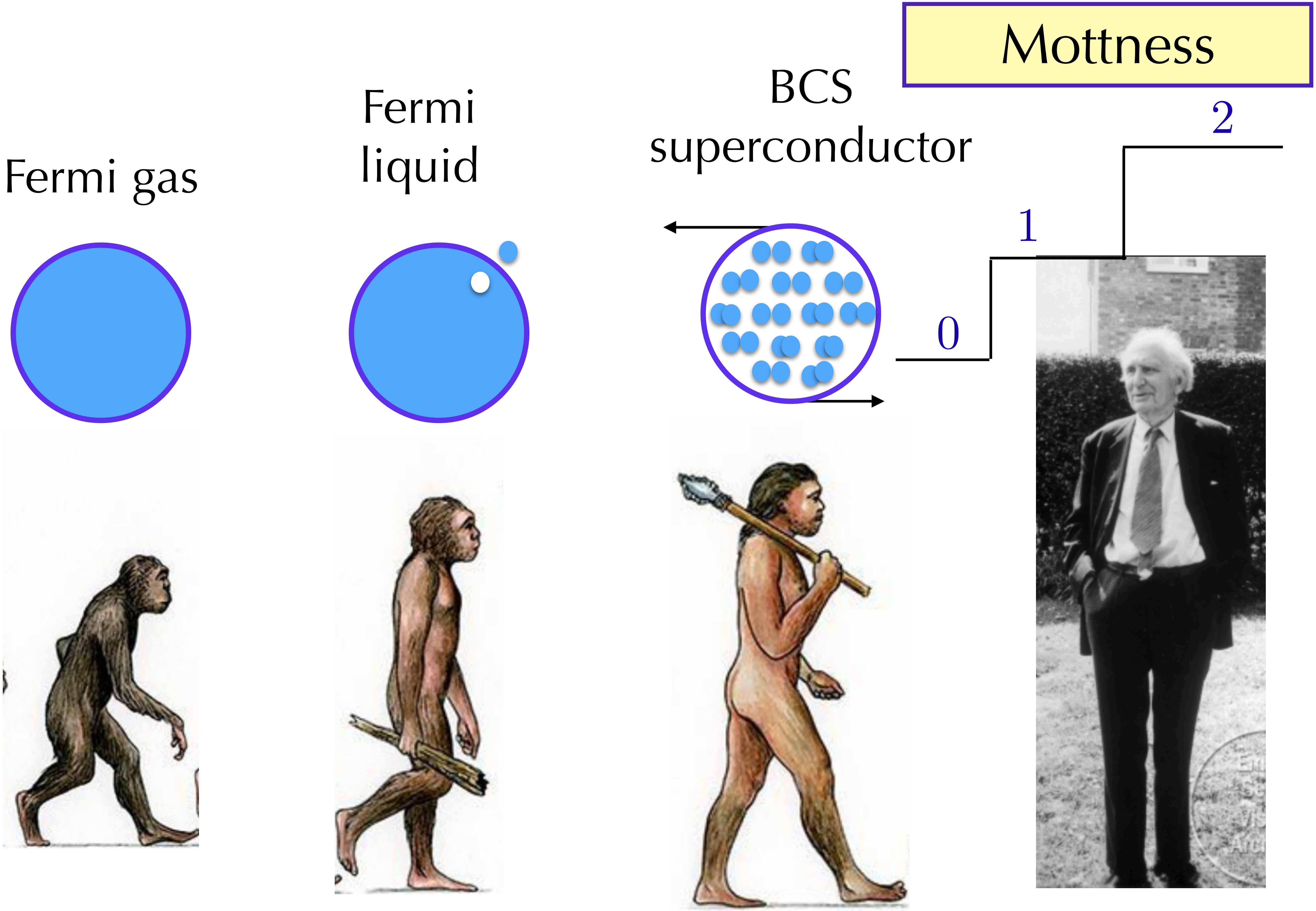


what does the HK model leave out??

$$[H_t, H_U] \neq 0$$



dynamical spectral weight transfer



Mottness
Fermi gas
Fermi
liquid
BCS
superconductor
0
1
2

Superconductivity?

Cooper Instability

$$H = H_{\mathrm{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k\in\Omega_0} \alpha_k b_k^\dagger |\mathrm{GS}\rangle$$

$$\langle n_{k\sigma}\rangle = 0$$

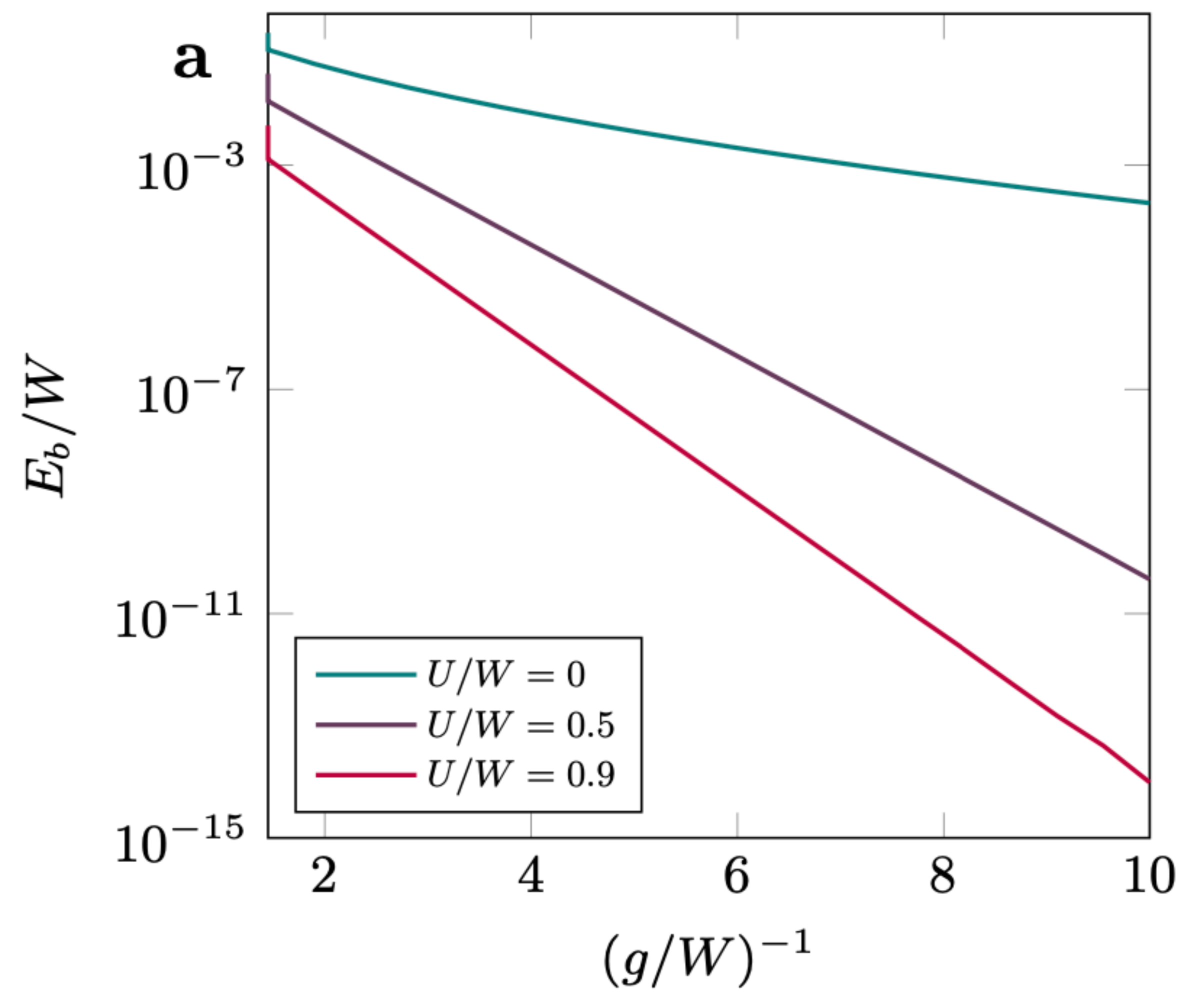
$$E_b = \langle \mathrm{GS}|\mathrm{H}|\mathrm{GS}\rangle - \langle\psi|\mathrm{H}|\psi\rangle \lessgtr 0$$

$$1 = -\frac{g}{L^d}\sum_{k\in\Omega_0} \frac{\langle 1 - n_{k\uparrow} - n_{-k\downarrow}\rangle}{E - 2\xi_k - U\langle n_{k\downarrow} + n_{-k\uparrow}\rangle}$$

$$1 = -g\int_{\mu}^{W/2} d\epsilon \frac{\rho(\epsilon)}{E - 2\epsilon + 2\mu},$$

Cooper Instability

$$E_b = -E \sim W(1-(U/W)^2)e^{-\pi W\sqrt{1-(U/W)^2}/g}$$



Pair Susceptibility

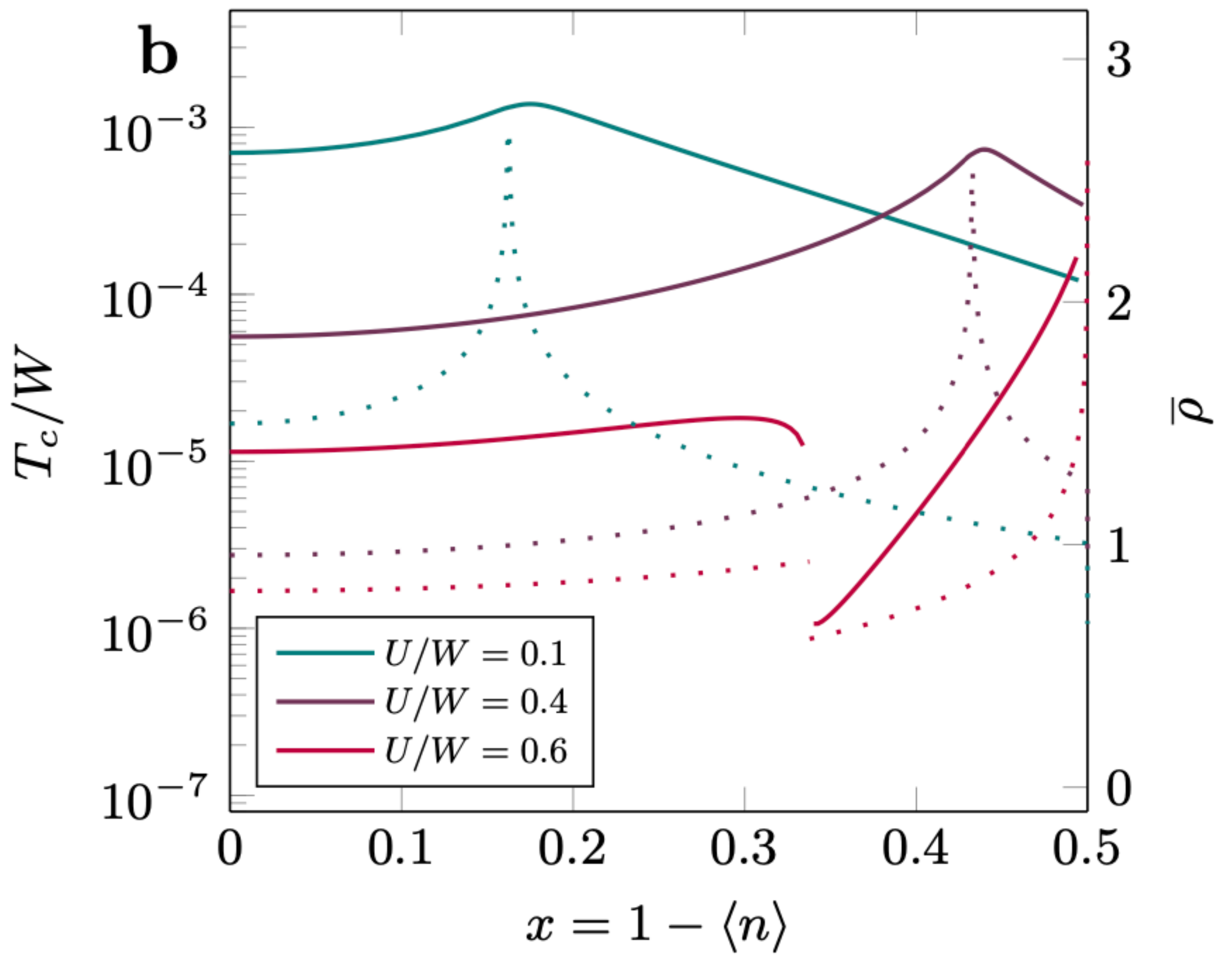
$$\chi(i\nu_n) \equiv \frac{1}{L^d}\int_0^\beta d\tau e^{i\nu_n\tau}\langle T\Delta(\tau)\Delta^\dagger\rangle_g$$

↓

$$= \frac{\chi_0}{1-g\chi_0}$$

↓ $g\chi_0 = 1$

solve for T_c



$W > U$

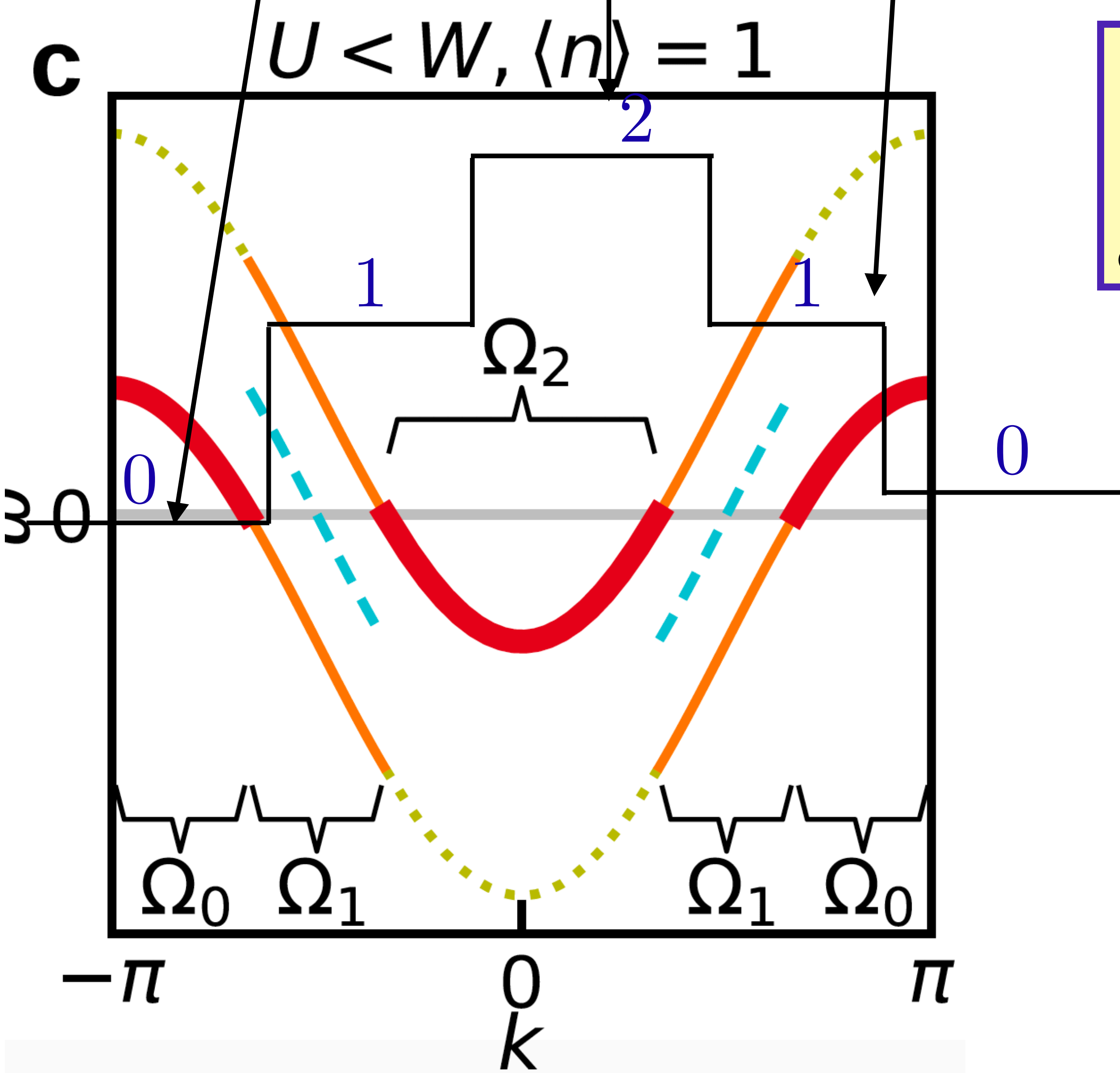
$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

variational wave function

$$|\psi_{\mathrm{BCS}}\rangle = \prod_k (u_k + v_k b_k^\dagger |0\rangle$$

$$|\psi_{\mathrm{BCS}}\rangle = \prod_{k>0} (u_k^2 + v_k^2 b_k^\dagger b_{-k}^\dagger + u_k v_k (b_k^\dagger + b_{-k}^\dagger))|0\rangle$$

$$|\psi\rangle = \prod_{k>0}\left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}}(b_k^\dagger + b_{-k}^\dagger)\right)|0\rangle$$



HK
generalization

three variational parameters

$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation

$$1 = \frac{g}{W}\sinh^{-1}\left(\frac{W-U}{2\Delta}\right) + \frac{g}{W}\sinh^{-1}\left(\frac{U}{2\Delta}\right)$$

$\downarrow$ $\Delta \ll U, W$

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

gap/T_c ratio

$$\Delta = (W-U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

$$T_c = (W-U)^{4/5} U^{1/5} \frac{e^{\gamma}}{\pi} e^{-\frac{4}{5}\frac{W}{g}}.$$

$$\lim_{g \to 0} \frac{\Delta}{T_c} \to \infty$$

non-BCS superconductivity

Bogoliubov excitations

$$\gamma_{k\sigma}|\psi_{\mathrm{BCS}}\rangle = 0$$

$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c^{\dagger}_{-k\overline{\sigma}}$$

PYHons excitations

$$\gamma^{l}_{k\sigma} \propto \sqrt{2} x_k \zeta^{\dagger}_{k\sigma} - \sigma z_k \zeta_{-k\overline{\sigma}}$$

$$\gamma^{u}_{k\sigma} \propto z_k \eta^{\dagger}_{k\sigma} - \sigma \sqrt{2} y_k \eta_{-k\overline{\sigma}}$$

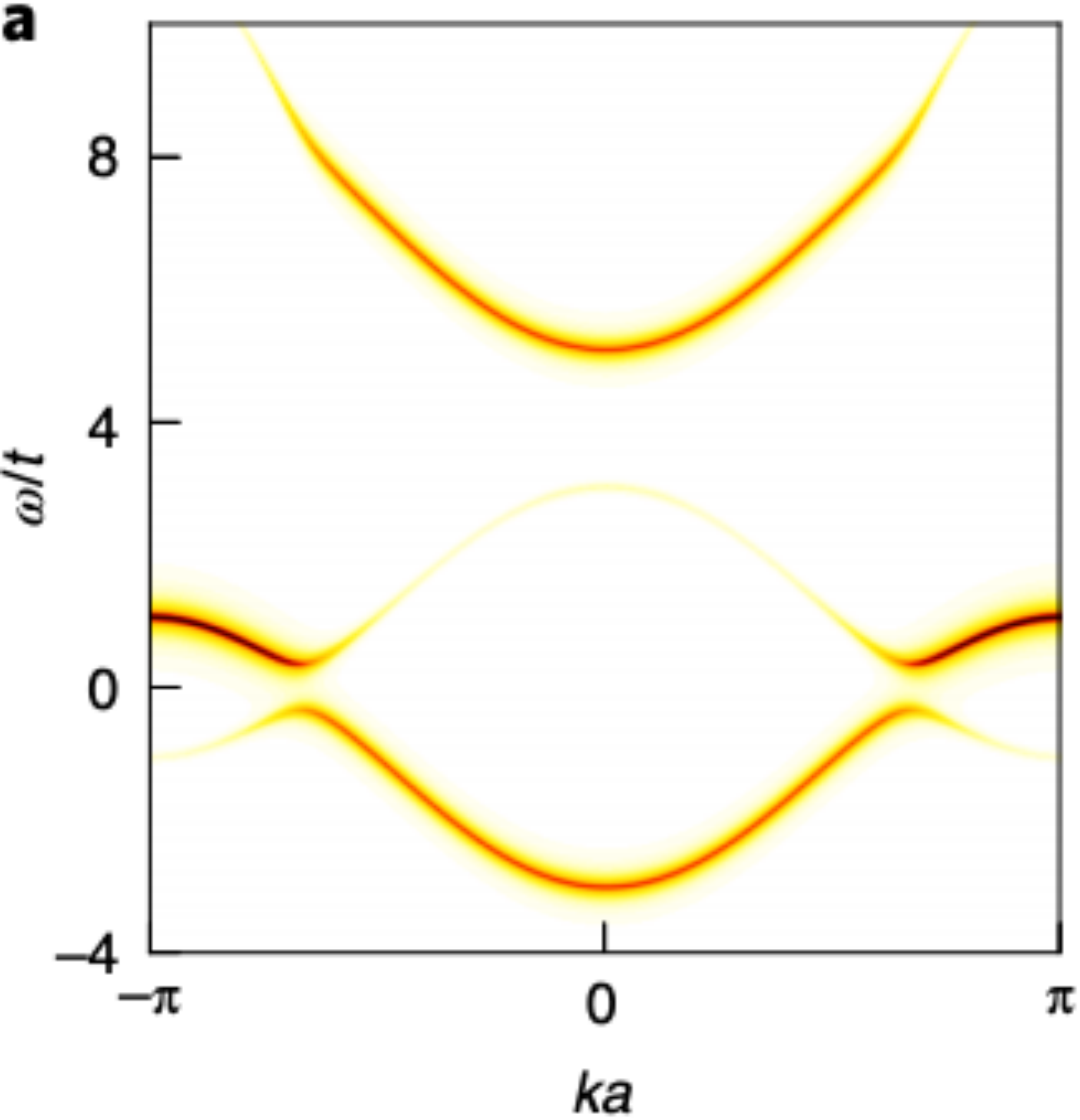
Excitation spectrum

$$\gamma_{k\sigma}^{u/l}|\psi\rangle = 0$$

$$\langle\psi|\gamma_{k\sigma}^{u/l} H \gamma_{k\sigma}^{u/l})^\dagger|\psi\rangle = \langle\psi|H|\psi\rangle + E_k^{u/l}$$

$$E_k^{u/l} = \sqrt{{\xi_k^{u/l}}^2 + \Delta^2}$$

superconductivity affects both bands!



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$

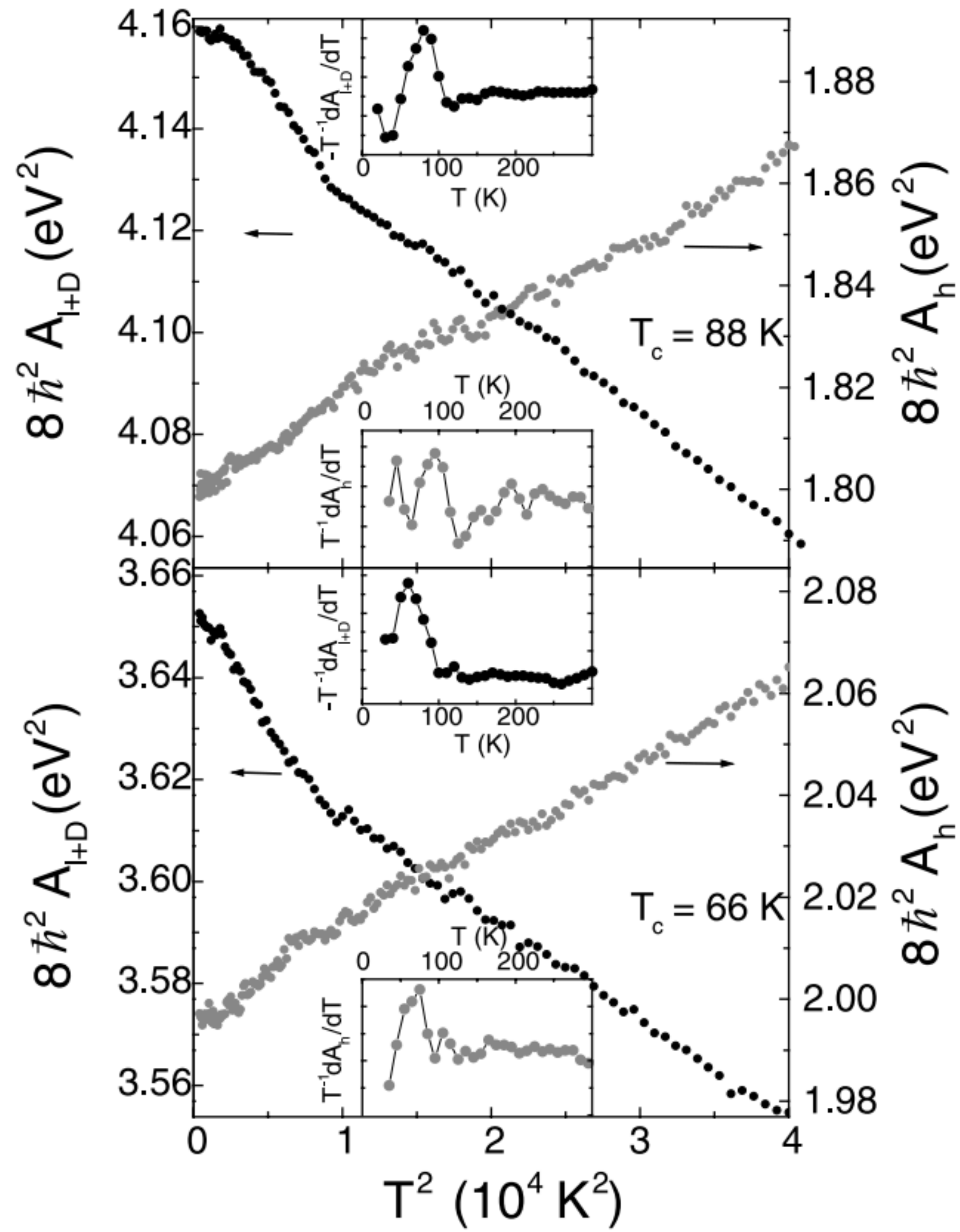
H. J. A. Molegraaf[1], C. Presura[1], D. van der Marel[1,*], P. H. Kes[2], M. Li[2]
+ See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 cm^{-1}$$

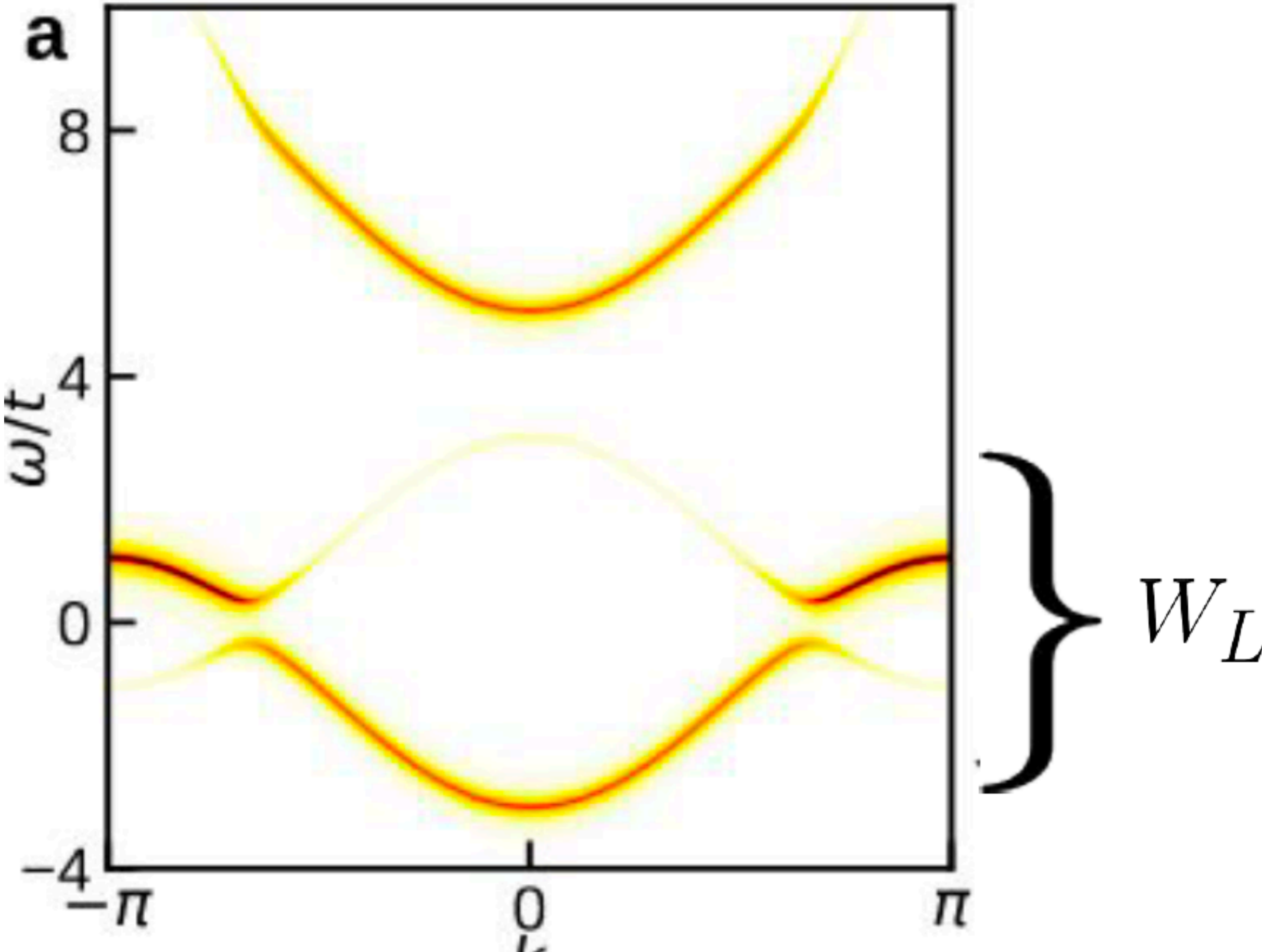
$$\frac{\Delta A_l}{A_l} \propto 3\%$$



condensation energy

Optical data are reported on a spectral weight transfer over a broad frequency range of $Bi_2Sr_2CaCu_2O_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least 2×10^4 cm^{-1}, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing


a
8
4
0
−4
ω/t
−π
0
π
k
W_L

why?

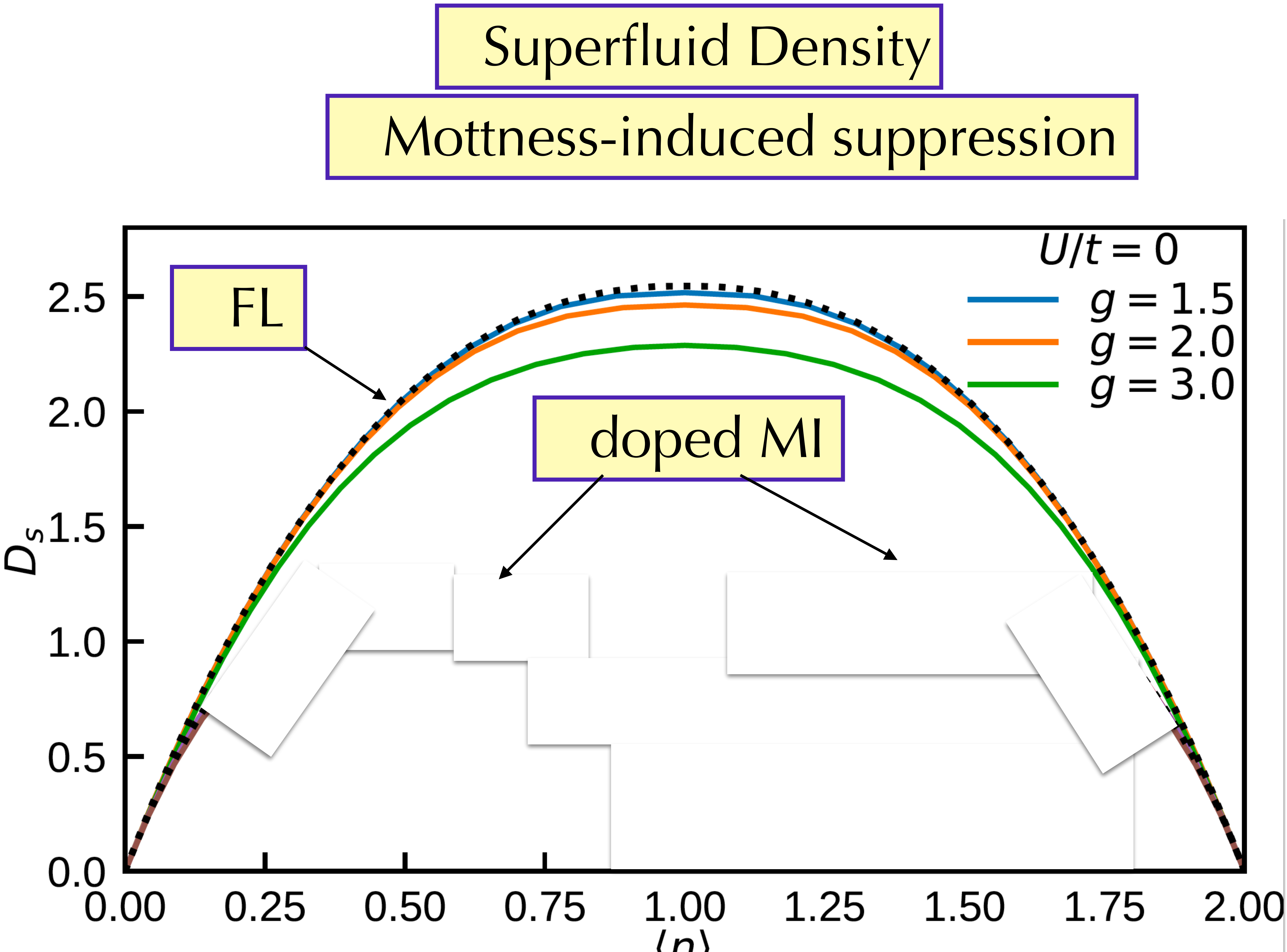
$$H = H_{\mathrm{HK}} + H_p$$

$$[H_{\mathrm{HK}}, H_p] \neq 0$$

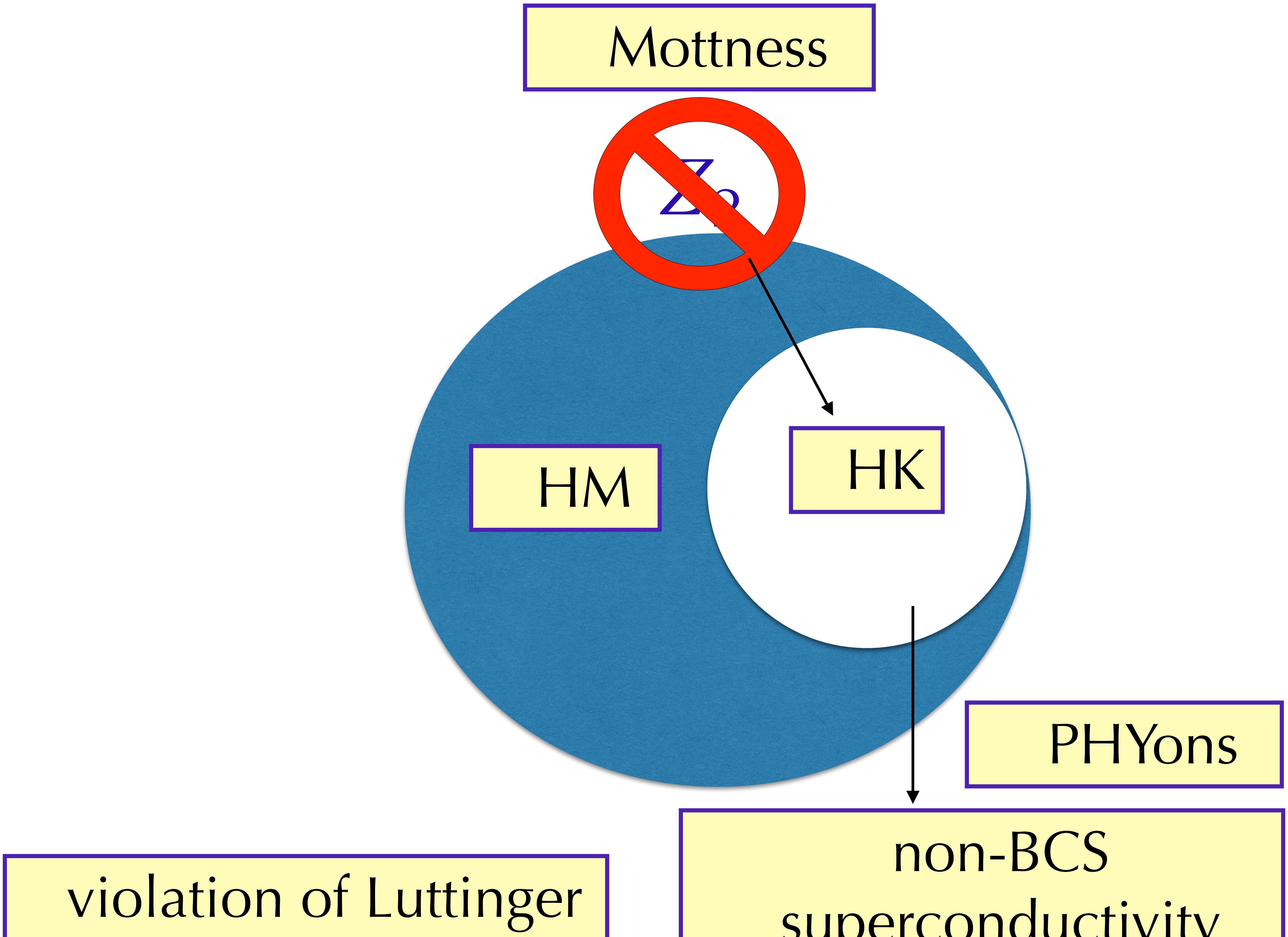
↓

dynamical spectral weight transfer

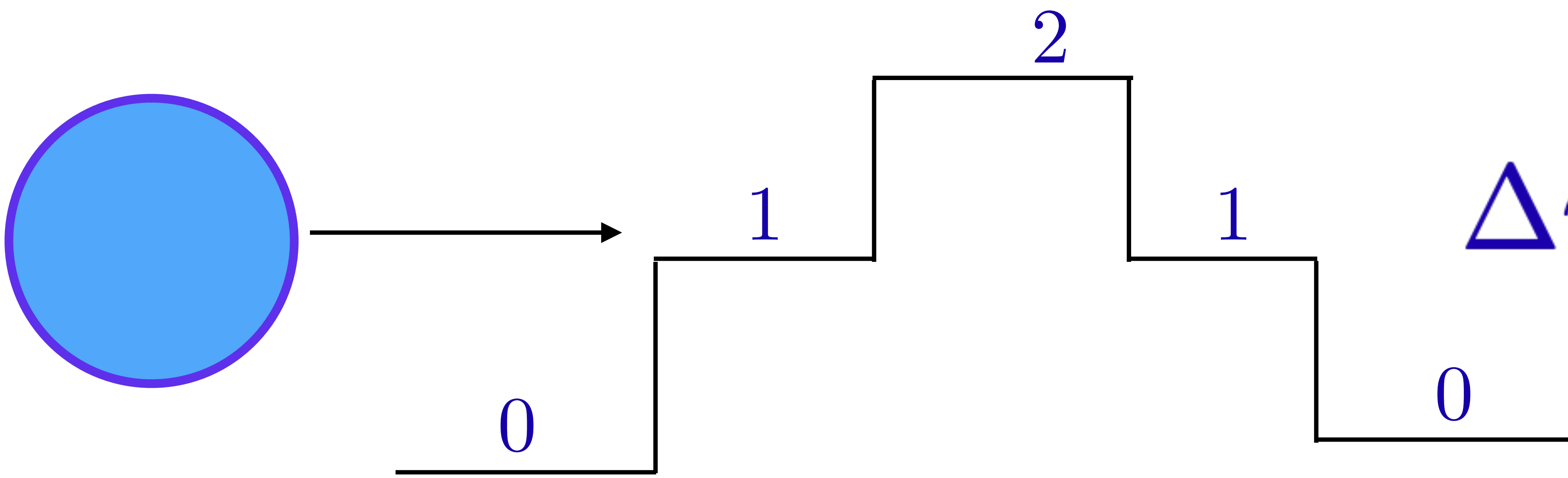
is this the general mechanism of the color change?



Superfluid Density
Mottness-induced suppression
FL
doped MI
U/t = 0
g = 1.5
g = 2.0
g = 3.0
D_s
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
$\langle n \rangle$


Mottness
HM
HK
PHYons
non-BCS superconductivity
violation of Luttinger

HK and EFL?



$$\Delta n = 1$$

other hand, we have

tween two regions with different $n(\mathbf{k})$ carries a Fermi surface with anomaly coefficient m given by the difference Δn. In a Fermi gas, $n(\mathbf{k})$ has a natural interpretation: it

$$\frac{2mq}{(2\pi)^2}\mathcal{V}_F = \rho \pmod 2. \tag{64}$$

$$\Delta n = m = 2$$

The extra factor of two takes into account the two possible spin values, and agrees (setting $m = 1$) with the usual result for spinful Fermi liquids. (Thus, if we had defined m with respect to the total charge as we did in Section VI, we would have found $m = 2$ for a spinful Fermi liquid/EFL).

?